6 LOWER PAIRS

Introduction

In the chapter of mechanisms and machines, basic mechanisms with their inversions were introduced. In this chapter, some more mechanisms of the lower pair category will be discussed. Lower pairs usually comprise turning (pivoted) and sliding pairs. Mechanisms with pivoted links are widely used in machines and the required movements of links are produced by using them in a variety of forms and methods. In this chapter, some of the more common mechanisms will be studied. Pantographs are used to copy the curves on reduced or enlarged scales. Some pivoted-link mechanisms are used to guide reciprocating parts either exactly or approximately in straight paths to eliminate the friction of the straight guides of the sliding pairs. However, these days, sliders are also being used to get linear motions.

An exact straight-line mechanism guides a reciprocating part in an exact straight line. On the other hand, an approximate straight-line mechanism is designed in such a way that the middle and the two extreme positions of the guided point are in a straight line and the intermediate positions deviate as little as possible from the line.

Although this chapter will be restricted to the more elementary aspects of the analysis of mechanisms, the possibilities of their use in the mechanisms and the machine of daily use can easily be glimpsed. Moreover, systematic design techniques are being developed so that these mechanisms can be used for accurate control of the processes and the machines being needed in modern technology.

6.1 PANTOGRAPH

A pantograph is a four-bar linkage used to produce paths exactly similar to the ones traced out by a point on the linkage. The paths so produced are, usually, on an enlarged or reduced scale and may be straight or curved ones.

The four links of a pantograph are arranged in such a way that a parallelogram *ABCD* is formed (Fig. 6.1). Thus, $AB = DC$ and $BC = AD$. If some point O in one of the links is made fixed and three other points P, Q and R on the other three links are located in such a way that *OPQR* is a straight line, it can be shown that the points P, Q and R always move parallel and similar to each other over any path, straight or curved. Their motions will be proportional to their distances from the fixed point.

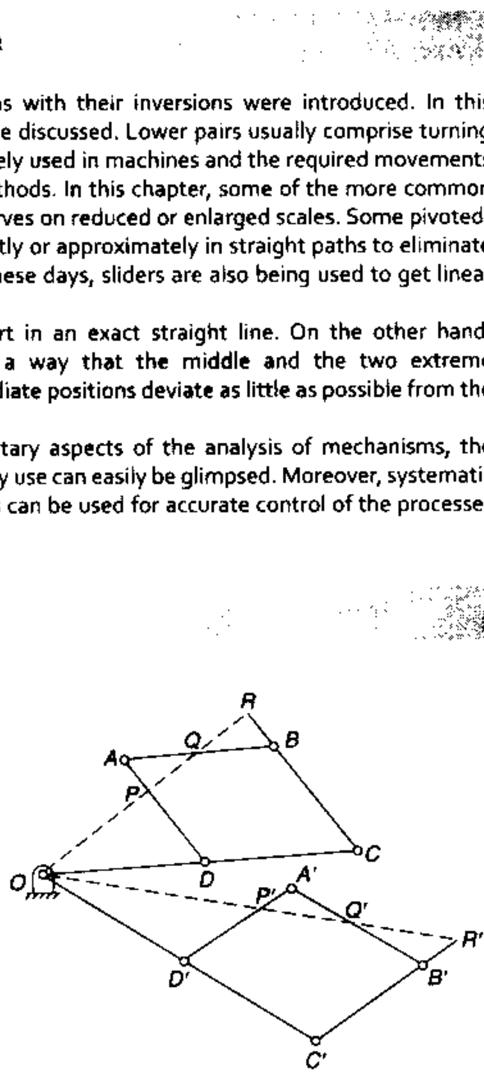

Fig. 6.1

Let O, P, Q and R lie on links CD, DA, AB and BC respectively. $ABCD$ is the initial assumed positions as shown in the figure.

Let the linkage be moved to another position so that A moves to A', B to B', and so on.

In ΔODP and OCR,

O, P and R lie on a straight line and thus OP and OR coincide.

$\angle DOP = \angle COR$ (common angle)

$\angle ODP = \angle OCR$ $(\because DP \mid\mid CR)$

Therefore, the Δs are similar and

$$\frac{OD}{OC} = \frac{OP}{OR} = \frac{DP}{CR} \quad \text{(i)}$$

Now, $A'B' = AB = DC = D'C'$

And $B'C' = BC = AD = A'D'$

Therefore, $A'B'C'D'$ is again a parallelogram.

In Δs $OD'P'$ and $OC'R'$,

$$\frac{OD'}{OC'} = \frac{OD}{OC} = \frac{DP}{CR} \quad \text{[from (i)]}$$

$$= \frac{D'P'}{C'R'}$$

and,

$\angle OD'P' = \angle OC'R'$ $(D'P' \parallel C'B'$ as $A'B'C'D'$ is a $\parallel$ gm)

Thus, the $\angle s$ are similar.

$\therefore \quad \angle D'OP' = \angle C'OR'$

or O, P' and R' lie on a straight line.

Now

$$\frac{OP}{OR} = \frac{OD}{OC} \quad \text{[from (i)]}$$

$$= \frac{OD'}{OC'}$$

$$= \frac{OP'}{OR'} \quad (\because \Delta s\ OD'P' \text{ and } OC'R' \text{ are similar})$$

This shows that as the linkage is moved, the ratio of the distances of P and R from the fixed point remains the same, or the two points are displaced proportional to their distances from the fixed point. This will be true for all the positions of the links. Thus, P and R will trace exactly similar paths.

Similarly, it can also be proved that P and Q trace similar paths. Thus, P, Q and R trace similar paths when the linkage is given motion.

6.2 STRAIGHT-LINE MECHANISMS

1. Paucellier Mechanism

A Paucellier mechanism consists of eight links (Fig. 6.2) such that,

$$OA = OQ; \qquad AB = AC$$

and $$BP = PC = CQ = QB$$

OA is the fixed link and OQ is a rotating link. It can be proved that as the link OQ moves around O, P moves in a straight line perpendicular to OA. All the joints are pin-jointed.

Since $BPCQ$ is a rhombus,

QP always bisects the angle BQC,

i.e.,

$$\angle 1 = \angle 2 \qquad \text{(i)}$$

in all the positions

Also, in Δs AQC and AQB,

AQ is common,

$AC = AB$

$QC = QB$

$\therefore$ Δs are congruent in all positions.

or $$\angle 3 = \angle 4 \qquad \text{(ii)}$$

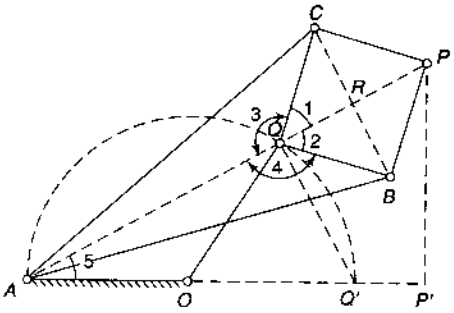

Fig. 6.2

Adding (i) and (ii),

$\angle 1 + \angle 3 = \angle 2 + \angle 4$

But $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360°$

$\therefore$ $\angle 1 + \angle 3 = \angle 2 + \angle 4 = 180°$

or A, Q and P lie on a straight line.

Let PP' be the perpendicular on AO produced.

Δs AQQ' and APP' are similar because $\angle 5$ is common and $\angle AQQ' = \angle AP'P = 90°$

$$\therefore \quad \frac{AQ}{AP'} = \frac{AQ'}{AP}$$

or
$$\begin{aligned} AQ'.AP' &= (AQ)(AP) \\ &= (AR - RQ)(AR + RP) \\ &= (AR - RQ)(AR + RQ) \qquad \ldots (RP = RQ) \\ &= (AR)^2 - (RQ)^2 \\ &= [(AC)^2 - (CR)^2] - [(CQ)^2 - (CR)^2] \end{aligned}$$

or
$$AP' = \frac{(AC)^2 - (CQ)^2}{AQ'}$$

= constant, as AC, CQ and AQ' are always fixed

This means that the projection of P and AQ produced is constant for all the configurations.

Thus, PP' is always a normal to AO produced or P moves in a straight line perpendicular to AO.

Example 6.1 *Figure 6.3(a) shows the link MAC which oscillates on a fixed centre A. Another link OQ oscillates on the centre O. The links AB and AC are equal. Also BP = PC = CQ = QB. Locate the position of O such that,*

(i) P moves in a straight line

(ii) P moves in a circle with centre A

(iii) P moves in a circle with centre at OA produced

(iv) P moves in a circle with centre O and Q moves in a straight line

(modify the lengths of links if necessary)

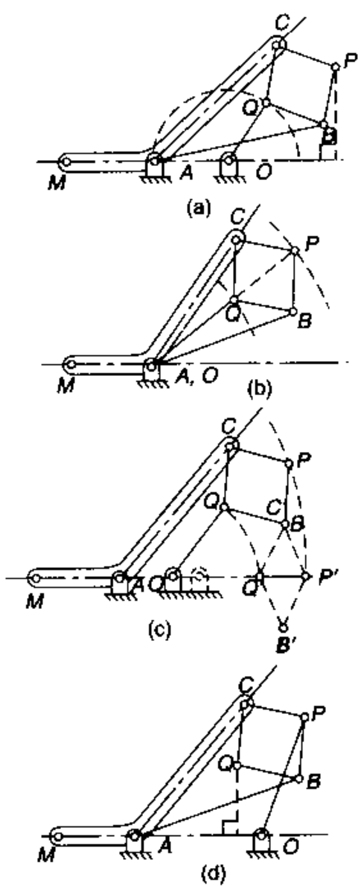

Fig. 6.3

Solution

(i) As in the Paucellier mechanism, O is located by drawing a straight line through A and perpendicular to the motion of P such that $AO = OQ$ [Fig. 6.3(a)].

(ii) If O is made to coincide with A, AQ would be equal to OQ. Thus, Q and P will be fixed on AP. Q will rotate about A and thus P will also rotate in a circle about A with AP as the radius [Fig. 6.3(b)].

(iii) From the above two cases, it can be observed that in (i) P moves in a circle with the centre at infinity on OA produced and in (ii) P moves in circle with the centre at A. Thus, if P is to move in a circle with the centre in-between A and infinity on OA produced, O must lie in-between O and A or in other words OQ should be greater than OA [Fig. 6.3(c)].

(iv) The mechanism will be similar to the Paucellier mechanism. P is to be joined with O by a link so that P moves in a circle about O and $OA = OP$. The lengths can be modified in two ways [Fig. 6.3(d)].

(a) OA is increased and OA and OP are made equal.

(b) Lengths AB and AC are reduced in such a way that $OA = OP$.

2. Hart Mechanism

A Hart mechanism consists of six links as shown in Fig. 6.4 such that

$$AB = CD; \quad AD = BC \quad \text{and } OE = OQ$$

OE is the fixed link and OQ, the rotating link. The links are arranged in such a way that $ABDC$ is a trapezium (AC parallel to BD). Pins E and Q on the links AB and AD respectively, and the point P on the link CB are located in such a way that

$$\frac{AE}{AB} = \frac{AQ}{AD} = \frac{CP}{CB} \qquad \text{(i)}$$

It can be shown that as OQ rotates about O, P moves in a line perpendicular to EO produced.

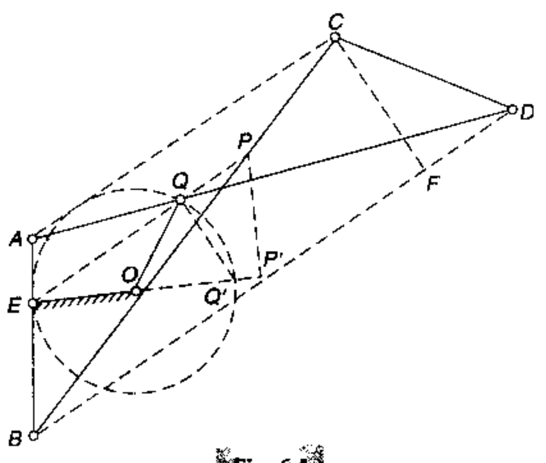

Fig. 6.4

In ΔABD $\dfrac{AE}{AB} = \dfrac{AQ}{AD}$ (Given)

Therefore, EQ is parallel to BD and thus parallel to AC.

In ΔABC $\dfrac{AE}{AB} = \dfrac{CP}{CB}$ (Given)

Therefore, EP is parallel to AC and thus parallel to BD.

Now, EQ and EP are both parallel to AC and BD and have a point E in common; therefore, EQP is a straight line.

Δs AEQ and ABD are similar ($\because EQ \parallel BD$).

$$\therefore \quad \frac{EQ}{BD} = \frac{AE}{AB} \text{ or } EQ = BD \times \frac{AE}{AB} \qquad \text{(ii)}$$

Δs BEP and BAC are similar ($\because EP \parallel AC$).

$$\therefore \quad \frac{EP}{AC} = \frac{BE}{BA} \text{ or } EP = AC \times \frac{BE}{AB} \qquad \text{(iii)}$$

Δs EQQ' and $EP'P$ are similar, because $\angle QEQ'$ or $\angle PEP'$ is common and $\angle EQQ' = \angle QP'P = 90°$.

$$\therefore \quad \frac{EQ}{EP'} = \frac{EQ'}{EP}$$

or $$EQ' \times EP' = EQ \times EP$$

$$= \left(BD \times \frac{AE}{AB} \right)\left(AC \times \frac{BE}{AB} \right) \qquad \text{[from (ii) and (iii)]}$$

or

$$EP' = \frac{AE \times BE}{(EQ')(AB)^2}[(BD)(AC)]$$

$$= \frac{AE \times BE}{(EQ')(AB)^2}\left[(BF + FD)(BF - FD)\right]$$

$$= \frac{AE \times BE}{(EQ')(AB)^2}\left[(BF)^2 - (FD)^2\right]$$

$$= \frac{AE \times BE}{(EQ')(AB)^2}\left[\{(BC)^2 - (CF)^2\} - \{(CD)^2 - (CF)^2\}\right]$$

$$= \frac{AE \times BE}{(EQ')(AB)^2}\left[(BC)^2 - (CD)^2\right]$$

= constant, as all the parameters are fixed.

Thus, EP' is always constant. Therefore, the projection of P on EO produced is always the same point or P moves in a straight line perpendicular to EO.

Example 6.2 ***A circle with EQ' as diameter has a point Q on its circumference. P is a point on EQ produced such that if Q turns about E, $EQ.EP$ is constant. Prove that the point P moves in a straight line perpendicular to EQ'.***

Solution Let PP' be perpendicular to EQ' produced (Fig. 6.5).

For any position of Q on the circumference of the circle with diameter EQ', Δs EQQ' and $EP'P$ are similar ($\angle QEQ'$ is common and $\angle EQQ' = \angle EP'P = 90°$).

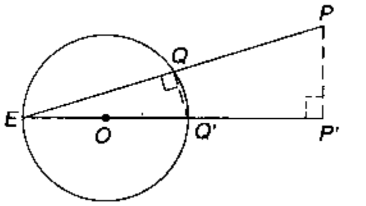

Fig. 6.5

$$\therefore \quad \frac{EQ}{EQ'} = \frac{EP'}{EP}$$

or $\quad EQ'.\,EP' = EQ.\,EP$

or $\quad EP' = \frac{EQ.EP}{EQ'}$

= constant, as EQ' is fixed and EQ. EP = constant

Thus, EP' will be constant for all positions of Q. Therefore, the location of P' is fixed which means that P moves in a straight line perpendicular to EQ'.

3. Scott–Russel Mechanism

A Scott-Russel mechanism consists of three movable links; OQ, PS and slider S which moves along OS. OQ is the crank (Fig. 6.6). The links are connected in such a way that

$$QO = QP = QS$$

It can be proved that P moves in a straight line perpendicular to OS as the slider S moves along OS.

As $QO = QP = QS$, a circle can be drawn passing through O, P and S with PS as the diameter and Q as the centre.

Now, O lies on the circumference of the circle and PS is the diameter. Therefore, $\angle POS$ is a right angle. This is true for all the positions of S and is possible only if P moves in a straight line perpendicular to OS at O.

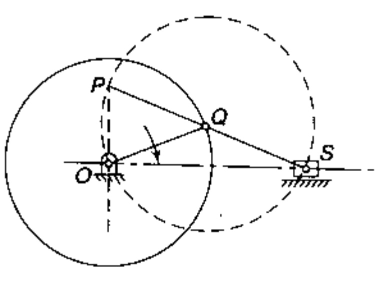

Fig. 6.6

Note that in such a mechanism, the path of P is through the joint O which is not desirable. This can be avoided if the links are proportioned in a way that QS is the mean proportional between OQ and QP, i.e.,

$$\frac{OQ}{QS} = \frac{QS}{QP}$$

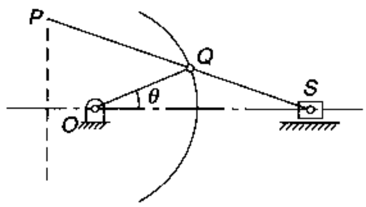

Fig. 6.7

However, in this case P will approximately traverse a straight line perpendicular to OS and that also for small movements of S or for small values of the angle θ (Fig. 6.7). A mathematical proof of this, being not simple, is omitted here. However, by drawing the mechanism in a number of positions, the fact can be verified.

Usually, this is known as the *modified Scott–Russel mechanism.*

4. Grass–Hopper Mechanism

This mechanism is a derivation of the modified Scott–Russel mechanism in which the sliding pair at S is replaced by a turning pair. This is achieved by replacing the slider with a link AS perpendicular to OS in the mean position. AS is pin-jointed at A (Fig. 6.8).

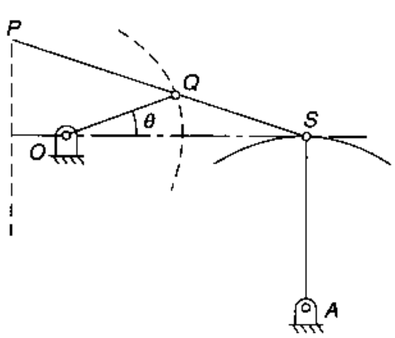

Fig. 6.8

If the length AS is large enough, S moves in an approximated straight line perpendicular to AS (or in line with OS) for small angular movements. P again will move in an approximate straight line if QS is the mean proportional between OQ and QP, i.e.,

$$\frac{OQ}{QS} = \frac{QS}{QP}$$

Example 6.3 *In a Grass–Hopper mechanism shown in Fig. 6.9, $OQ = 80$ mm, $SQ = 120$ mm and $SP = 300$ mm. Find the magnitude of the vertical force at P necessary to resist a torque of 100 N.m applied to the link OQ when it makes angles of 0°, 10° and 20° with the horizontal.*

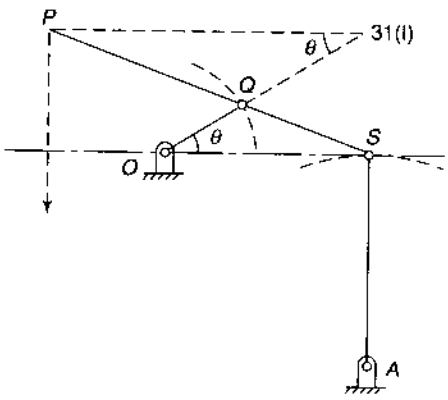

Fig. 6.9

Solution

$OQ = 80$ mm

$QS = 120$ mm

$QP = 300 - 120 = 180$ mm

$$\frac{OQ}{QS} = \frac{QS}{QP}, \text{ i.e., } \frac{80}{120} = \frac{120}{180}$$

As the condition for the dimensions of the Grass–Hopper mechanism is satisfied, P moves in an approximate straight line for small angles of OQ with the horizontal.

Now

$$F_p \times v_p = T_q \times \omega_q$$

$$F_p = \frac{T_p \omega_q}{v_p} = \frac{T_q}{v_p} \frac{v_q}{OQ} \qquad \text{(i)}$$

Locate the I-centre (instantaneous centre) of the link SP. It is at 31 as the directions of motions of points P and Q on it are known.

$$\frac{v_q}{v_p} = \frac{IQ}{IP} = \frac{OQ}{OS}$$

($\because$ Δs IQP and OQS are similar)

(i) becomes $F_p = \dfrac{T_q}{OS}$

When $\theta = 0°$, $OS = 80 + 120 = 200$ mm

$$F_p = \frac{100}{0.2} = 500 \text{ N}$$

When $\theta = 10°$,

$$OS = 80 \cos 10° + \sqrt{(120)^2 - (80 \sin 10°)^2}$$

$= 198$ mm

$$F_p = \frac{100}{0.198} = \underline{505.05 \text{ N}}$$

When $\theta = 20°$,

$$OS = 80 \cos 20° + \sqrt{(120)^2 - (80 \sin 20°)^2}$$

$= 192$ mm

$$F_p = \frac{100}{0.192} = \underline{520.8 \text{ N}}$$

As the angle θ increases, P moves in only approximate straight line and thus the calculations for F_p are not exact.

5. Watt Mechanism

It is a very simple mechanism. It has four links OQ, OA, QB and AB. OQ is the fixed link. Links OA and QB can oscillate about centres O and Q respectively. It is seen that if P is a point on the link AB such that $PA/PB = QB/OA$, then for small oscillations of OA and QB, P will trace an approximately straight line. This has been shown in Fig. 6.10 for three positions.

In earlier times, the mechanism was used by Watt to guide the piston, as it was difficult to machine plane surfaces.

6. Tchebicheff Mechanism

It consists of four links OA, QB, AB and OQ (fixed) as shown in Fig. 6.11. The links OA and QB are equal and crossed. P, the mid-point of AB, is the tracing point. The proportions of the links are taken in such a way that P, A and B lie on vertical lines when on extreme positions, i.e., when directly above O or Q.

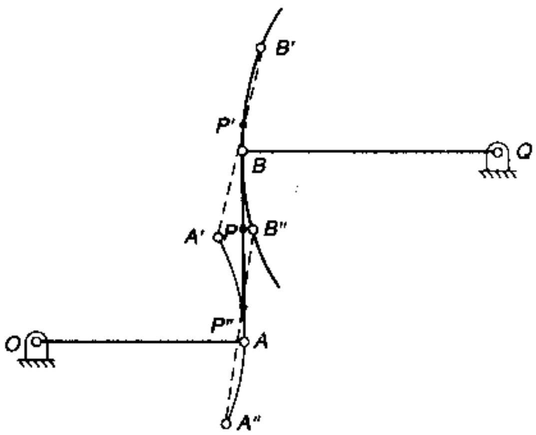

Fig. 6.10

Let $AB = 1$ unit

$OA = QB = x$ units

and $OQ = y$ units

When AB is on the extreme left position, A and B assume the positions A' and B', respectively.

In $\Delta OQB'$,

$$(QB')^2 - (OQ)^2 = (OB')^2$$

$$(QB)^2 - (OQ)^2 = (OB')^2 \qquad (OB' = OB)$$

$$x^2 - y^2 = (OA' - A'B')^2$$

$$= (x-1)^2$$

$$= x^2 - 2x + 1$$

or $$2x - 1 = y^2$$

$$x = \frac{y^2+1}{2} \qquad \text{(i)}$$

In ΔOAC,

$$(OA)^2 - (AC)^2 = (OC)^2$$

$$(OA)^2 - (OP')^2 = (AP')^2$$

$$(OA)^2 - (OA' - A'P')^2 = (PP' + AP)^2$$

$$x^2 - \left(x - \frac{1}{2}\right)^2 = \left(\frac{y}{2} + \frac{1}{2}\right)^2$$

or $$x^2 - \left(x^2 + \frac{1}{4} - x\right) = \frac{y^2}{4} + \frac{1}{4} + \frac{y}{2}$$

$$x = \frac{y^2}{4} + \frac{y}{2} + \frac{1}{2} \qquad \text{(ii)}$$

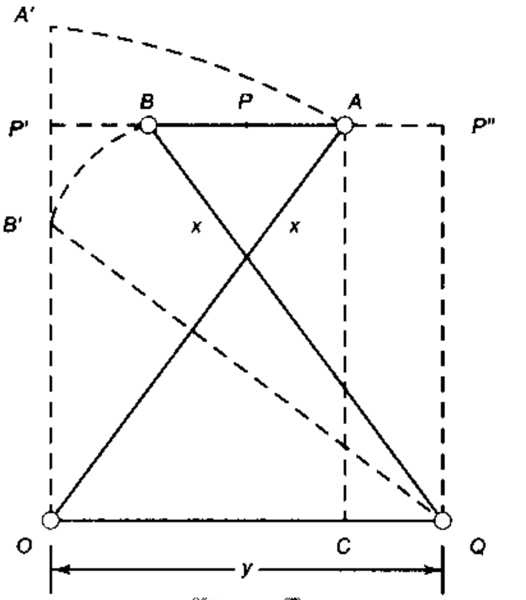

Fig. 6.11

From Eqs (i) and (ii),

$$\frac{y^2}{2} + \frac{1}{2} = \frac{y^2}{4} + \frac{y}{2} + \frac{1}{2}$$

$$\frac{y^2}{4} = \frac{y}{2}$$

or $$y = 2$$

and $$x = \frac{y^2+1}{2} = 2.5$$

Thus, $AB : OQ : OA = 1:2:2.5$

This ratio of the links ensures that P moves approximately in a horizontal straight line parallel to OQ.

7. Kempe's Mechanism

This mechanism consists of two identical mechanisms *ABCDEF* and *A′B′C′D′EF′*. All pairs are turning pairs as shown in Fig. 6.12.

The ratios of the links are

$AF = AC = 2(EC = ED = EF)$
$= 4(BD = BC)$

and

$A'F' = A'C' = 2(EC' = ED' =$
$EF') = 4(B'D' = B'C')$

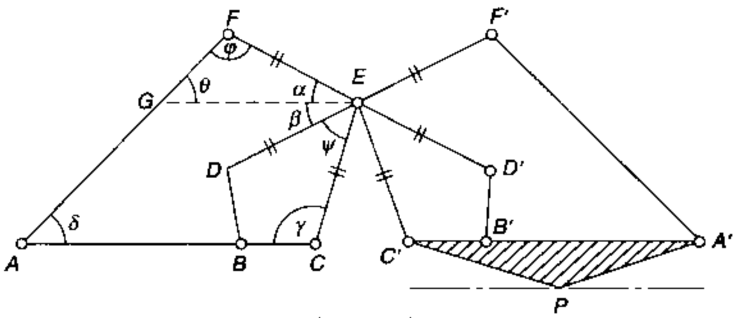

Fig. 6.12

Links *DEF′* and *D′EF* are rigid links having turning pairs at *E* and at the ends.

It can be proved that if *ABC* is a fixed horizontal link, *A′B′C′* also remains horizontal (in line with *ABC*) and thus any point on the link such as *P* will move in a horizontal straight line.

Quadrilaterals *ACEF* and *EDBC* are similar because,

$$\frac{AC}{ED} = \frac{CE}{DB} = \frac{EF}{BC} = \frac{FA}{CE} = 2$$

and $\angle\varphi = \angle\gamma$

(angles between corresponding sides when two pairs of adjacent sides are equal in the quadrilateral *ACEF*)

$\therefore \quad \angle\delta = \angle\psi$ (corresponding angles of two quadrilaterals)

Draw $EG \parallel CA$

In $\Delta\, EFG$,

$\angle\alpha + \angle\varphi + \angle\theta = \pi$

or $\angle\alpha = \pi - \angle\varphi - \angle\theta$

$= \pi - \angle\varphi - \angle\delta$ $\quad (\because EG \parallel CA)$

$= \pi - \angle\gamma - \angle\psi$ $\quad$ (i)

But as *CE* cuts *EG* and *CA*, two parallel lines,

$\angle\gamma + \angle\psi + \angle\beta = \pi$

or $\angle\gamma = \pi - \angle\psi - \angle\beta$

$\therefore$ from (i) $\angle\alpha = \pi - (\pi - \angle\psi - \angle\beta) - \angle\psi$

$= \angle\beta$

Thus, for all configurations, $\angle\alpha = \angle\beta$, i.e., inclination of *ED* and *EF* is same to *EG* or *CA* and the two identical parts of the mechanism always remain symmetrical.

Hence, if *ABC* is a fixed horizontal link, *A′B′C′* also remains horizontal in line with *ABC* and any point *P* on it traces a horizontal path.

8. Parallel Linkages

If the opposite links of a four-link mechanism are made equal, the linkage will always form a parallelogram. The following types of parallel linkages are used universally.

Parallel Ruler As shown in Fig. 6.13, in a parallel ruler, all the horizontal links have the same length, i.e.,

$$AB = CD = EF = GH = IJ$$

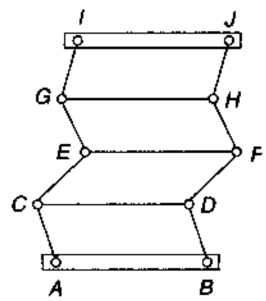

Fig. 6.13

The lengths of the opposite links of each parallelogram should also be equal, i.e.,

$$AC = BD,\ CE = DF,\ EG = FH \text{ and } GI = HJ$$

It can be seen that any number of parallelograms can be used to form this ruler. The dimensions of the mechanism ensure that IJ moves parallel to AB.

Lazy Tongs In this mechanism (Fig. 6.14), O is pin-jointed and is a fixed point. The point A slides in the vertical guides while all other points are pin-jointed. All the links are of equal length. As A moves vertically, P will move in an approximate horizontal line. The use of such a mechanism can be made in supporting a bulb (of a table lamp) or telephone, etc.

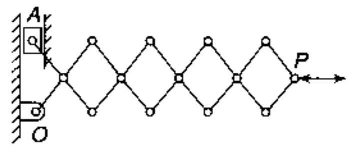

Fig. 6.14

Universal Drafting Machine In such a mechanism (Fig. 6.15), two parallelograms of the links are formed.

$$AB = CD \quad \text{and} \quad AC = BD$$

The link AB is fixed.

As $ABDC$ is a parallelogram, CD always remains parallel to AB. C and D are pin-jointed to a disc D_1. Thus, the disc D_1 can have translatory motion in a plane but not angular motions.

EF is another link on the disc D_1 pin-jointed at the ends E and F. As the orientation of the disc D_1 is fixed, the direction of EF is also fixed.

Also,

$$EG = FH \quad \text{and} \quad EF = GH$$

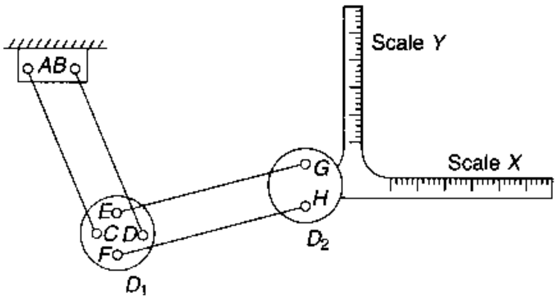

Fig. 6.15

Thus, the direction of GH is always parallel to EF or there is no angular movement of the disc D_2. Therefore, scales X and Y will always be along the horizontal and the vertical directions.

A universal drafting machine is extensively used as a substitute for T-square and set-square.

6.3 ENGINE INDICATORS

An *indicator* of a reciprocating engine is an instrument that keeps the graphical record of pressure inside the cylinder during the piston stroke.

An indicator consists of an *indicator cylinder* with a *piston*. The indicator cylinder is connected to the engine cylinder. Thus, varying pressure of the gas or steam is communicated to the indicator piston, the displacement of which is constrained by a spring to get a direct measure of the gas or the steam pressure. The displacement is recorded by a pencil on paper, wrapped on a drum, to a suitable scale with the help of a straight-line mechanism.

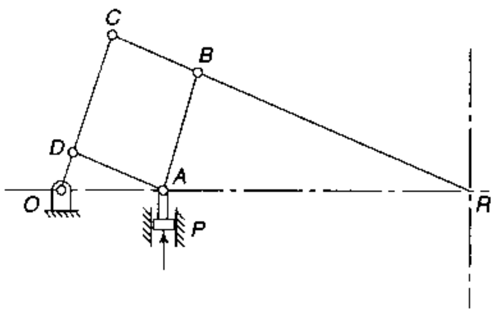

Fig. 6.16

The following are the usual types of indicators:

1. Simplex Indicator

This indicator employs the mechanism of a pantograph. As shown in Fig. 6.16, O is the fixed pivot whereas $ABCD$ is a parallelogram formed by the four links. R is a point on the link CB produced to trace the path of A or P (movement of piston). Also, refer to Fig. 6.1.

P moves in a vertical straight line within the guides or the indicator cylinder. Its movement is controlled by the steam or the gas pressure to be measured. Thus, R also moves in a vertical straight line recording the variation of pressure with the help of a pencil recorder.

Example 6.4 ***Design a pantograph for an indicator to be used to obtain the indicator diagram of an engine. The distance between the fixed point and the tracing point is 180 mm. The indicator diagram should be three times the gas pressure inside the cylinder of the engine.***

Solution

Refer Fig. 6.16,

$$OR = 180 \text{ mm and } \frac{OR}{OA} = 3 \text{ (given)}$$

or $$\frac{180}{OA} = 3$$

or $$OA = 60 \text{ mm}$$

The relationship of the different arms of a simplex indicator is as follows:

$$\frac{OR}{OA} = \frac{OC}{OD} = \frac{CR}{CB} = 3$$

Choose convenient dimensions of OD and DA. Let these be 30 mm and 50 mm respectively.

Thus, as $ABCD$ is to be a parallelogram and the above relation is to be fulfilled, the other dimensions will be

$$OC = 30 \times 3 = 90 \text{ mm}$$
$$CR = 50 \times 3 = 150 \text{ mm}$$

Construct the diagram as follows:

1. Locate D by making arcs of radii 30 mm and 50 mm with centres O and A respectively.
2. Produce OD to C such that $OC = 90$ mm.
3. Join CR.
4. Draw AB parallel to OC.

Thus, the required pantograph is obtained.

2. Crosby Indicator

This indicator employs a modified form of the pantograph. The mechanism has been shown in Fig. 6.17.

To have a vertical straight line motion of R, it must remain in line with O and P, and also the links OC and PB must remain approximately parallel.

As P lies on the link 3 and R on 5, locate the I-centres 31 and 51. If the directions of velocities of any two points on a link are known, the I-centre can be located easily which is the intersection of the perpendiculars to the directions of velocities at the two points.

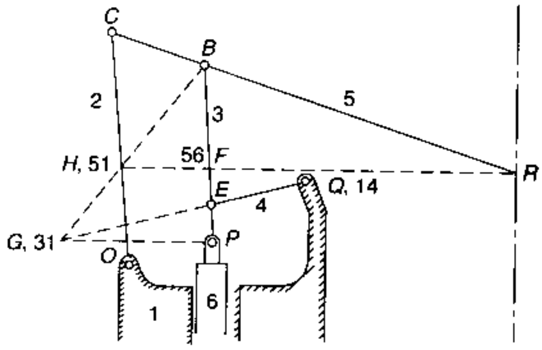

Fig. 6.17

First, locate 31 as the directions of velocities of P and E on the link 3 are known.

- The direction of velocity of P is vertical. Therefore, 31 lies on a horizontal line through P.
- The direction of velocity of E is perpendicular to QE. Therefore, 31 lies on QE (or QE produced).

The intersection of QE produced with the horizontal line through P locates the point 31.

Thus, the link 3 has its centre of rotation at 31 (link 1 is fixed) and the velocity of any point on the link is proportional to its distance from 31, the direction being perpendicular to a line joining the point with the I-centre.

To locate 51, the directions of velocities of B and C are known.

- The direction of velocity of B is $\perp$ to $31 - B$. Therefore, 51 lies on $31 - B$.
- The direction of velocity of C is $\perp$ to OC. Therefore, 51 lies on OC.

Thus, 51 can be located.

Now, the link 5 has its centre of rotation at 51. The direction of velocity of the point R on this link will be perpendicular to 51-R. To have a vertical motion of R, it must lie on a horizontal line through 51.

The ratio of the velocities of R and P is given by,

$$\frac{v_r}{v_p} = \frac{v_r}{v_b} \cdot \frac{v_b}{v_p} \qquad (B \text{ is common to 3 and 5})$$

$$= \frac{51-R}{51-B} \cdot \frac{31-B}{31-P}$$

$$= \frac{51-R}{51-B} \cdot \frac{51-B}{51-F} \qquad (\because \Delta\text{s } BPG \text{ and } BFH \text{ are similar})$$

$$= \frac{51-R}{51-F}$$

$$= \frac{CR}{CB} \qquad (\because \Delta\text{s } CRH \text{ and } BRF \text{ are similar})$$

$$= \text{constant}$$

This shows that the velocity or the displacement of R will be proportional to that of P.

Alternatively, locate the I-centre 56 by using Kennedy's theorem. It will be at the point F (the intersection of lines joining I-centres 16, 15 and 35, 36, not shown in the figure).

First, consider this point 56 to lie on the link 6. Its absolute velocity is the velocity of 6 in the vertical direction (1 being fixed).

Now, consider the point 56 to lie on the link 5. The motion of 5 is that of rotation about 51 (1 being fixed). Thus, velocity of R on the link 5 can be found as the velocity of 56, another point on the same link is known.

$$\frac{v_r}{v_f} = \frac{51-R}{51-F}$$

or

$$= \frac{51-R}{51-F} \qquad (v_f = v_p)$$

$$= \frac{CR}{CB}$$

3. Thomson Indicator

A Thomson indicator employs a Grass–Hopper mechanism $OCEQ$. R is the tracing point which lies on CE produced as shown in Fig. 6.18.

The best position of the tracing point R is obtained as discussed below:

Locate the I-centres 31 and 51 as in case of a Crosby indicator. The directions of velocities of two points C and E on the link 5 are known; therefore, first locate the I-centre 51.

- The direction of velocity of C is $\perp$ to OC. Therefore 51 lies on OC.
- The direction of velocity of E is $\perp$ to QE, Therefore, 51 lies on QE (or QE produced).

Thus, 51 can be located.

Now, the directions of velocities of two points B and P on the link 3 are known.

The direction of velocity of B is $\perp$ to 51-B, (B is on the link 5 also)

Therefore, 31 lies on the line 51-B.

The direction of velocity of P is vertical,

Therefore, 31 lies on a horizontal line through P.

Thus, 31 can be located.

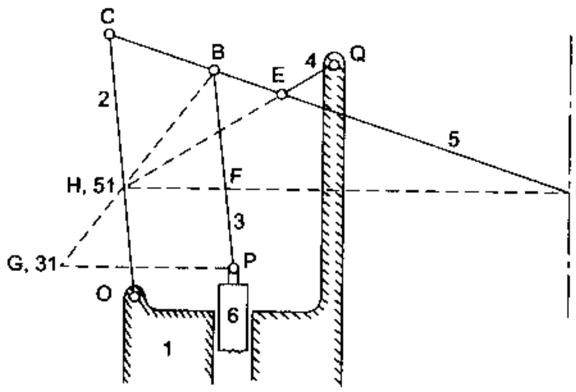

Fig. 6.1[illegible]

As R is to move in a vertical direction, it must lie on a horizontal line through the I-centre of the link 5 on which the pointer lies.

Similar to the case of a Crosby indicator, the velocity ratio is given by,

$$\frac{v_r}{v_p} = \frac{CR}{CB} = \text{constant}$$

Therefore, the velocity or the displacement of R is proportional to that of P.

It is to be remembered that since OC and PB do not remain parallel for all positions, R moves in an approximate vertical line. However, the variations are negligible.

Alternatively, the I-centre 56 can be located by using Kennedy's theorem. It will be at the point F.

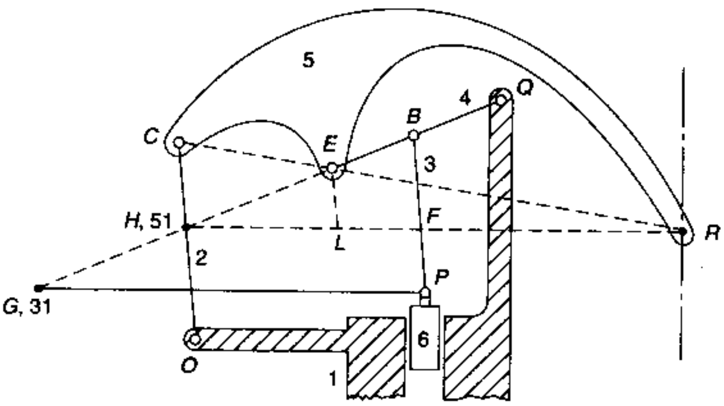

Fig. 6.1[illegible]

4. Dobbie McInnes Indicator

This indicator is similar to a Thomson indicator, the difference being that the link 3 is pivoted to a point in the link 4 instead of a point on the link 5. Thus, the motion of the indicator piston is imparted to the link 4.

The indicator is shown in Fig. 6.19.

Locate the I-centre 31 as before.

Locate R by finding the intersection of CE and a horizontal line through 51.

Locate I-centre 31as usual.

Now

$$\frac{v_r}{v_p} = \frac{v_r}{v_e} \times \frac{v_e}{v_b} \times \frac{v_b}{v_p}$$

$$= \frac{51-R}{51-E} \cdot \frac{QE}{QB} \cdot \frac{31-B}{31-P}$$

$$= \frac{51-R}{51-E} \cdot \frac{QE}{QB} \cdot \frac{51-B}{51-F} \quad (\because \Delta s\ BPG \text{ and } BFH \text{ are similar})$$

$$= \frac{51-R}{51-E} \cdot \frac{QE}{QB} \cdot \frac{51-E}{51-L} \quad (\because \Delta s\ BFH \text{ and } ELH \text{ are similar})$$

$$= \frac{51-R}{51-L} \cdot \frac{QE}{QB}$$

$$= \frac{CR}{CE} \cdot \frac{QE}{QB}$$

= constant

This expression also gives approximately the ratio of the displacement of R to that of P.

AUTOMOBILE STEERING GEARS

When an automobile takes turns on a road, all the wheels should make concentric circles to ensure that they roll on the road smoothly and there is a line contact between the tyres and the surface of the path, preventing the excess wear of tyres. This is achieved by mounting the two front wheels on two short axles, known as *stub axles*. The stub axles are pin-jointed with the main front axle which is rigidly attached to the rear axle. Thus, the steering is affected by the use of front wheels only.

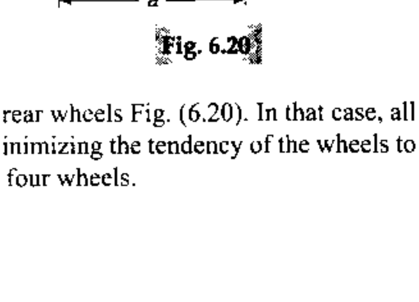

Fig. 6.20

When the vehicle is making a turn towards one side, the front wheel of that side must swing about the pin through a greater angle than the wheel of the other side. The ideal relation between the swings of the two wheels would be if the axes of the stub axles, when produced, intersect at a point I on the common axis of the two rear wheels Fig. (6.20). In that case, all the wheels of the vehicle will move about a vertical axis through I, minimizing the tendency of the wheels to skid. The point I is also the instantaneous centre of the motion of the four wheels.

Let θ and φ = angles turned by the stub axles

l = wheel base

w = distance between the pivots of front axles

Then,

$$\cot\varphi = \frac{PT}{TI} \text{ and } \cot\theta = \frac{QT}{TI}$$

$$\cot\varphi - \cot\theta = \frac{PT - QT}{TI} = \frac{PQ}{TI} = \frac{w}{l} \qquad (6.1)$$

This is known as the *fundamental equation of correct gearing*. Mechanisms that fulfil this fundamental equation are known as *steering gears*.

6.5 TYPES OF STEERING GEARS

There are two main types of steering gears:

1. Davis steering gear
2. Ackermann steering gear

A *Davis steering gear* has sliding pairs which means more friction and easy wearing. The gear fulfils the fundamental equation of gearing in all the positions. However, due to easy wearing it becomes inaccurate after some time.

An *Ackermann steering gear* has only turning pairs and thus is preferred. Its drawback is that it fulfils the fundamental equation of correct gearing at the middle and the two extreme positions and not in all positions.

Davis Steering Gear

A Davis steering gear shown in Fig. 6.21(a) consists of two arms PK and QL fixed to the stub axles PC and QD to form two similar bell-crank levers CPK and DQL pivoted at P and Q respectively. A cross link or track arm AB, constrained to slide parallel to PQ, is pin-jointed at its ends to two sliders. The sliders S_1 and S_2 are free to slide on the links PK and QL respectively.

During the straight motion of the vehicle, the gear is in the mid-position with equal inclination of the arms PK and QL with PQ.

As the vehicle turns right, the cross-arm AB also moves right through a distance x from the mid-position as shown in Fig.6.21(b). The bell-crank levers assume the positions $C'PK'$ and $D'QL'$.

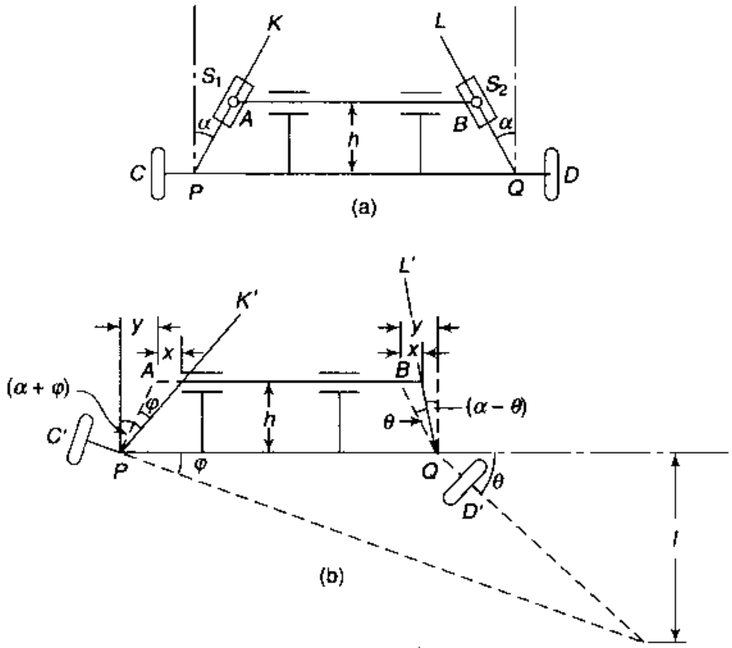

Fig. 6.21

Let h = vertical distance between AB and PQ

$$\tan(\alpha-\theta)=\frac{y-x}{h}$$

$$\frac{\tan\alpha-\tan\theta}{1+\tan\alpha\tan\theta}=\frac{y-x}{h}$$

$$\frac{\frac{y}{h}-\tan\theta}{1+\frac{y}{h}\tan\theta}=\frac{y-x}{h} \qquad \left(\tan\alpha=\frac{y}{h}\right)$$

$$\frac{y-h\tan\theta}{xh+y\tan\theta}=\frac{y-x}{h}$$

$$(y-h\tan\theta)h=(h+y\tan\theta)(y-x)$$

$$yh - h^2 \tan\theta = hy - hx + y^2 \tan\theta - xy\tan\theta$$

$$hx = (y^2 - xy + h^2)\tan\theta$$

$$\tan\theta = \frac{hx}{y^2 - xy + h^2}$$

Also, $\tan(\alpha + \varphi) = \frac{y + x}{h}$

and it can be proved that $\tan\varphi = \frac{hx}{y^2 + xy + h^2}$

For correct steering action, $\cot\varphi - \cot\theta = \frac{w}{l}$

or

$$\frac{y^2 + xy + h^2}{hx} - \frac{y^2 - xy + h^2}{hx} = \frac{w}{l}$$

or $$\frac{2xy}{hx} = \frac{w}{l}$$

or $$\frac{y}{h} = \frac{w}{2l}$$

or $$\tan\alpha = \frac{w}{2l} \quad (6.2)$$

The usual value of w/l is between 0.4 to 0.5 and that of α from 11 or 14 degrees.

Example 6.5 *The ratio between the width of the front axle and that of the wheel base of a steering mechanism is 0.44. At the instant when the front inner wheel is turned by 18°, what should be the angle turned by the outer front wheel for perfect steering?*

Solution

$w/l = 0.44 \qquad \theta = 18°$

As $\cot\varphi - \cot\theta = \frac{w}{l}$

$\therefore \quad \cot\varphi - \cot 18° = 0.44$

$\cot\phi = 0.44 + 3.078 = 3.518$

or $\phi = \underline{15.9°}$

Example 6.6 *The distance between the steering pivots of a Davis steering gear is 1.3 m. The wheel base is 2.75 m. What will be the inclination of the track arms to the longitudinal axis of the vehicle if it is moving in a straight path?*

Solution

$w = 1.3$ m $\qquad l = 2.75$ m

$$\tan\alpha = \frac{w}{2l} = \frac{1.3}{2 \times 2.75} = 0.236$$

$\therefore \quad \alpha = 13.3°$ or $13°\ 18'$

Example 6.7 *The track arm of a Davis steering gear is at a distance of 192 mm from the front main axle whereas the difference between their lengths is 96 mm. If the distance between steering pivots of the main axle is 1.4 m, determine the length of the chassis between the front and the rear wheels. Also, find the inclination of the track arms to the longitudinal axis of the vehicle.*

Solution

$w = 1.4$ m $\qquad h = 192$ mm $\qquad y = 96/2 = 48$ mm

$$\tan\alpha = \frac{y}{h} = \frac{48}{192} = 0.25$$

$\therefore \quad \alpha = \underline{14°}$

Also $\quad \tan\alpha = \frac{w}{2l}$

$\therefore \quad \tan 14° = \frac{1.4}{2l}$

or $\quad l = \underline{2.8\text{ m}}$

Ackermann Steering Gear

This steering gear consists of a four-link mechanism *PABQ* having four turning pairs.

As shown in Fig. 6.22(a), two equal arms *PA* and *QB* are fixed to the stub axles *PC* and *QD* to form two similar bell-crank levers *CPA* and *DQB* pivoted at *P* and *Q* respectively. A cross link *AB* is pin-jointed at the ends to the two bell-crank levers.

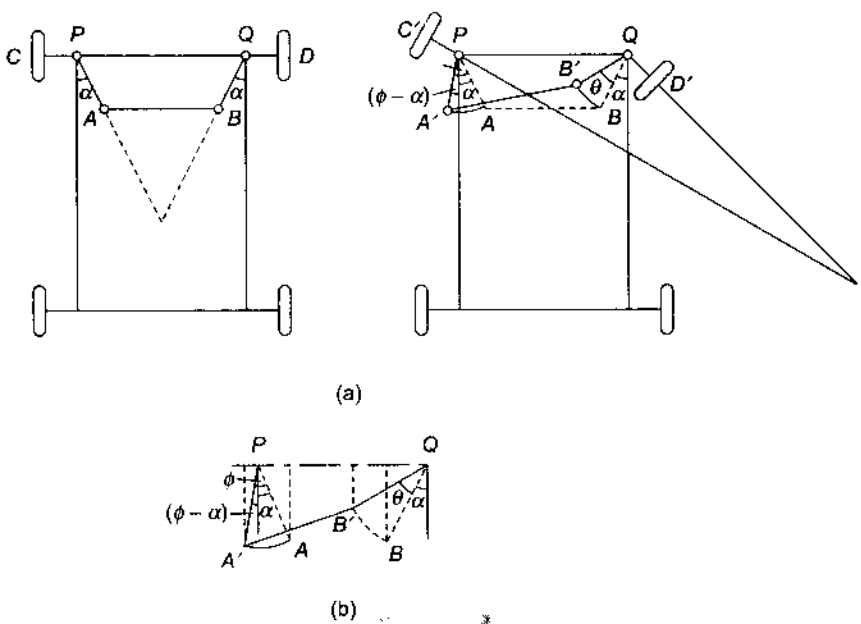

Fig. 6.22

During the straight motion of the vehicle, the gear is in the mid-position with equal inclination of the arms *PA* and *QB* with *PQ*. The cross link *AB* is parallel to *PQ* in this position.

An Ackermann gear does not fulfil the fundamental equation of correct gearing in all the positions but only in three positions. If the values of *PA*, *PQ* and the angle α are known, the mechanism can be drawn to a suitable scale in different positions. The angle ϕ can be noted for different values of θ. This angle ϕ found by drawing the gear may be termed as φ_a (φ actual).

Correct or theoretical values of φ corresponding to different values of θ, for the given values of w and l can be calculated from the relation for correct gearing, $\cot \varphi - \cot\theta = w/l$. The angle so obtained may be termed as φ_t (φ theoretical).

Comparing φ_a and φ_t, following observations are made:

1. For small values of θ, φ_a is marginally higher than φ_t
2. For larger values of θ, φ_a is lower than φ_t and the difference is substantial.

Thus, for larger values of θ or when the vehicle is taking a sharp turn, the wear of the tyres can be more due to slipping. However, to take sharp turns, the vehicle has to be slowed down, which reduces the wear of the tyres. Thus, the large difference between φ_a and φ_t does not affect much the life of tyres.

In an Ackermann gear, the instantaneous centre *I* does not lie on the rear axis but on a line parallel to the rear axis at an approximate distance of $0.3l$ above it.

Three positions of correct gearing are

1. when the vehicle moves straight,
2. when the vehicle moves at a correct angle to the right, and
3. when the vehicle moves at a correct angle to the left.

In all other positions, pure rolling is not possible due to slipping of the wheel.

Graphically, the two positions of the correct gearing are found by finding ($\cot \varphi - \cot \theta$) at different positions. The values that give the correct values of w/l ($w/l \approx 0.45$) correspond to correct gearing.

Determination of Angle α As mentioned above, if the values of PA, PQ and the angle α are known, the mechanism can be drawn to a suitable scale in different positions and the actual angle ϕ can be noted for different values of θ. The values of θ and ϕ matching with theoretical values provides the position of the vehicle for correct steering on the left and right. However, for initial design of the steering, usually angle α is obtained by assuming the steering in a position in which the projections of AB and $A'B'$ on PQ are equal [Fig.6.22(b)],

i.e.,

Projection of BB' on PQ = Projection of AA' on PQ

$$QB\,[\sin(\alpha+\theta)-\sin\alpha] = PA\,[\sin\alpha+\sin(\varphi-\alpha)]$$

or $$\sin(\alpha+\theta)-\sin\alpha = \sin\alpha+\sin(\varphi-\alpha) \qquad (PA = QB)$$

$$(\sin\alpha\cos\theta+\cos\alpha\sin\theta)-\sin\alpha = \sin\alpha+\sin\varphi\cos\alpha-\cos\varphi\sin\alpha$$

$$\sin\alpha(\cos\theta+\cos\varphi-2) = \cos\alpha(\sin\varphi-\sin\theta)$$

$$\frac{\sin\alpha}{\cos\alpha} = \frac{\sin\varphi-\sin\theta}{\cos\theta+\cos\varphi-2}$$

$$\tan\alpha = \frac{\sin\varphi-\sin\theta}{\cos\theta+\cos\varphi-2} \qquad (6.3)$$

where θ and φ are the values of angles for the correct gearing.

6.6 HOOKE'S JOINT

A Hooke's joint (Fig. 6.23), commonly known as a *universal joint*, is used to connect two non-parallel and intersecting shafts. It is also used for shafts with angular misalignment. A common application of this joint is in an automobile where it is used to transmit power from the gear box (of the engine) to the rear axle. The driving shaft rotates at a uniform angular speed whereas the driven shaft rotates at a continuously varying angular speed.

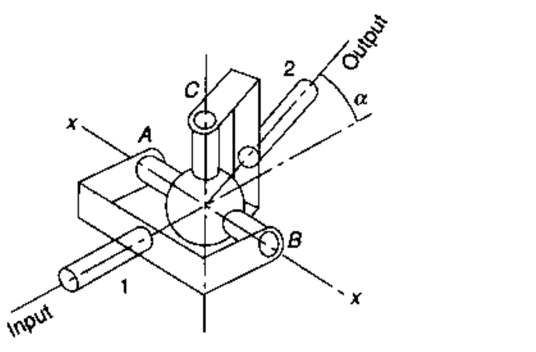

Fig. 6.23

The shafts 1 and 2 rotate in the fixed bearings. A complete revolution of either shaft will cause the other to rotate through a complete revolution in the same time, but with varying angular speed. Each shaft has a fork at its end. The four ends of the two forks are connected by a centre piece, the arms of which rest in the bearings, provided in the forks ends. The centre piece can be in the shape of a *cross*, *square* or *sphere* (having four pins or arms). The four arms of the cross are at right angles.

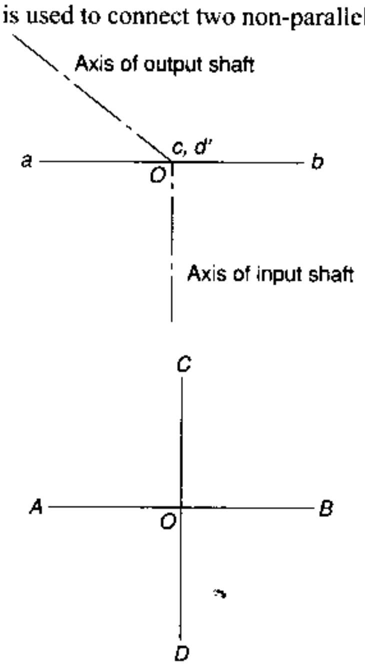

Fig. 6.24

Let two horizontal shafts, the axes of which are at an angle α, be connected by a Hooke's joint. If the joint is viewed along the axis of the shaft 1, the fork ends of this shaft will be A and B as shown in Fig. 6.24. C and D are the positions assumed by the fork ends of the shaft 2. The

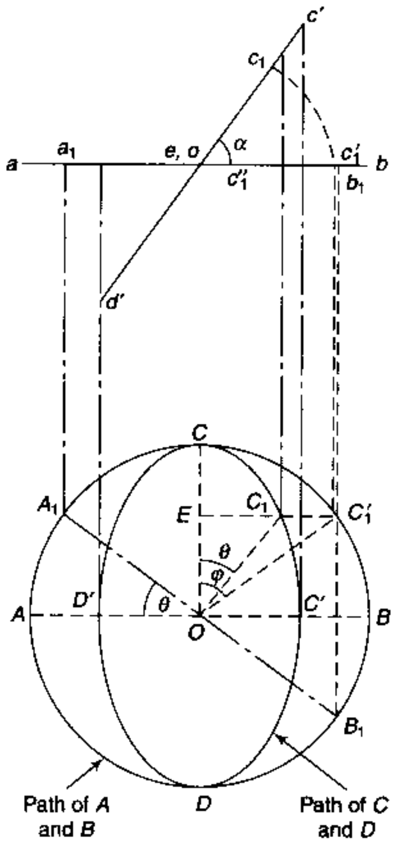

Fig. 6.25

axis of the shaft 1 is along the perpendicular to the plane of paper at O and that of the shaft 2 along OA. When viewed from top, c and d, projections of C and D coincide with that of O whereas a and b remain unchanged.

As the shaft 1 is rotated, its fork ends A and B, are rotated in a circle (Fig. 6.25). However, the fork ends C and D of the shaft 2 will move along the path of an ellipse, if viewed along the axis of the shaft 1. In the top view, the motion of the fork ends of the shaft 1 is along the line ab whereas that of the shaft 2 on a line $c'd'$ at an angle of α to ab.

Let the shaft 1 rotate through an angle θ so that fork ends assume the positions A_1 and B_1. Now, the angle moved by the shaft 2 would also be θ when viewing along the axis of the shaft 1. Let the fork end C take the position C_1. However, the true angle turned by the shaft 2 would be when it is viewed along its own axis. In Fig. 6.26, the front view of the joint is shown, when viewing along the axis of the shaft 2. Here, C and D move in a circle. The point C_1 lies on a circle at the same height as it is on the ellipse in Fig. 6.25. This gives the true angle φ turned by the shaft 2.

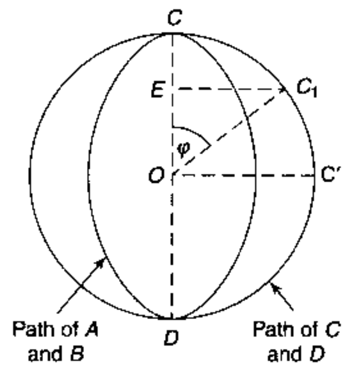

Fig. 6.26

Now,

$$\frac{\tan\varphi}{\tan\theta} = \frac{EC_1'/EO}{EC_1/EO} \qquad \text{(Fig. 6.25)}$$

$$= \frac{EC_1'}{EC_1}$$

$$= \frac{ec_1'}{ec_1''} \qquad \text{(Fig. 6.25 top view)}$$

$$= \frac{ec_1}{ec_1''}$$

$$= \frac{1}{ec_1''/ec_1}$$

$$= \frac{1}{\cos a}$$

or

$$\tan\theta = \cos\alpha\tan\varphi \qquad (6.4)$$

Angular Velocity Ratio

Let ω_1 = angular velocity of driving shaft $\left(= \frac{d\theta}{dt}\right)$

ω_2 = angular velocity of driving shaft $\left(= \frac{d\varphi}{dt}\right)$

Differentiating Eq. 6.4 with respect to time t,

$$\sec^2 \theta \frac{d\theta}{dt} = \cos\alpha \sec^2 \varphi \frac{d\varphi}{dt}$$

or

$$\frac{d\varphi / dt}{d\theta / dt} = \frac{\sec^2 \theta}{\cos\alpha \sec^2 \varphi}$$

$$\frac{\omega_2}{\omega_1} = \frac{1}{\cos^2 \theta \cos\alpha\, (1 + \tan^2 \varphi)}$$

$$= \frac{1}{\cos^2 \theta \cos\alpha \left(1 + \frac{\tan^2 \theta}{\cos^2 \alpha}\right)} \quad \ldots \quad \left(\tan\varphi = \frac{\tan\theta}{\cos\alpha}\right)$$

$$= \frac{1}{\cos^2 \theta \cos\alpha \left(1 + \frac{\sin^2 \theta}{\cos^2 \theta \cos^2 \alpha}\right)}$$

$$= \frac{\cos^2 \theta \cos^2 \alpha}{\cos^2 \theta \cos\alpha\, (\cos^2 \theta \cos^2 \alpha + \sin^2 \theta)}$$

$$= \frac{\cos\alpha}{\cos^2 \theta\, (1 - \sin^2 \alpha) + \sin^2 \theta}$$

$$= \frac{\cos\alpha}{\cos^2 \theta - \cos^2 \theta \sin^2 \alpha + \sin^2 \theta}$$

$$= \frac{\cos\alpha}{1 - \sin^2 \alpha \cos^2 \theta} \quad (6.5)$$

(i) $\frac{\omega_2}{\omega_1}$ is unity when $\frac{\cos\alpha}{1 - \sin^2 \alpha \cos^2 \theta} = 1$

or $\cos\alpha = 1 - \sin^2 \cos^2 \theta$

or $\cos^2 \theta = \frac{1 - \cos\alpha}{\sin^2 \alpha}$

$$= \frac{1-\cos\alpha}{1-\cos^2\alpha}$$

$$= \frac{1-\cos\alpha}{(1+\cos\alpha)(1-\cos\alpha)}$$

$$= \frac{1}{1+\cos\alpha}$$

$$= \frac{\sin^2\theta+\cos^2\theta}{1+\cos\alpha}$$

$$= \frac{\cos^2\theta\left(\dfrac{\sin^2\theta}{\cos^2\theta}+1\right)}{1+\cos\alpha}$$

or
$$\frac{\sin^2\theta}{\cos^2\theta}+1 = 1+\cos\alpha$$

or
$$\tan^2\theta = \cos\alpha$$

$$\tan\theta = \pm\sqrt{\cos\alpha} \qquad (6.6)$$

Thus, $\omega_2 = \omega_1$ or the velocities of the driven and the driving shafts are equal when the condition is fulfilled. This is possible once in all the four quadrants for particular values of θ if α is constant.

(ii) $\dfrac{\omega_2}{\omega_1}$ is minimum when the denominator of Eq. (6.5) is maximum,

i.e., $(1 - \sin^2\alpha\cos^2\theta)$ is maximum.

This is so when $\cos^2\theta$ is minimum,

or $\theta = 90°$ or $270°$

Then, $$\frac{\omega_2}{\omega_1} = \cos\alpha \qquad (6.7)$$

(iii) $\dfrac{\omega_2}{\omega_1}$ is maximum when the denominator of Eq. (6.5) is minimum,

i.e. $(1 - \sin^2\alpha\cos^2\theta)$ is minimum,

or
$$\theta = 0° \text{ or } 180°$$

and
$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-\sin^2\alpha} = \frac{\cos\alpha}{\cos^2\alpha} = \frac{1}{\cos\alpha} \qquad (6.8)$$

The variation in the speed of the driven shaft corresponding to the rotation of the driving shaft is shown in Fig. 6.27. Points '*e*' correspond to the angular displacements of the driving shaft when the angular velocity of the driven shaft is equal to that of the driving shaft. Points 'min' and 'max' correspond to the angular displacements of the driving shaft when the angular speeds of the driven shaft are the minimum and the maximum respectively.

Graphically, the variation of angular velocity of the driven shaft can be represented by an ellipse whereas that of the driving shaft, by a circle (Fig 6.28). Such a diagram is known as a *polar velocity diagram.*

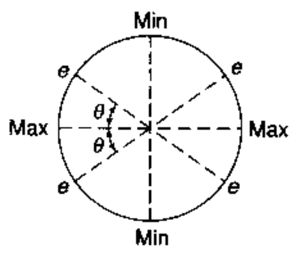

Fig. 6.27

Maximum variation of velocity of the driven shaft of its mean velocity

$$= \frac{\omega_{2\,max} - \omega_{2\,min}}{\omega_{mean}}$$

But ω_{mean} of the driven shaft is equal to the angular velocity ω_1 of the driving shaft as both the shafts complete one revolution in the same period of time.

$$\text{Maximum variation} = \frac{\omega_1 / \cos\alpha - \omega_1 \cos\alpha}{\omega_1} \quad (6.9)$$

$$= \frac{1-\cos^2\alpha}{\cos\alpha}$$

$$= \frac{\sin^2\alpha}{\cos\alpha}$$

$$= \tan\alpha \sin\alpha \quad (6.10)$$

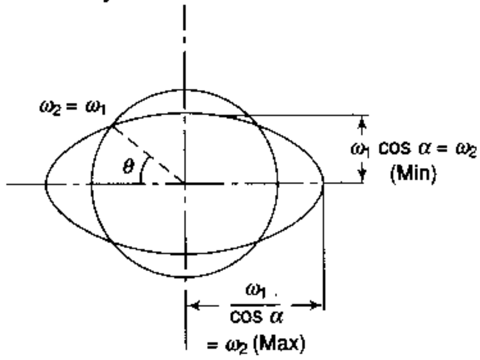

Fig. 6.2x

If α is small, i.e., the angle between the axes of the two shafts is small,

$\sin\alpha \approx \tan\alpha \approx \alpha$

Maximum variation $\approx \alpha^2$

(The mean speed ω is not equal to $\frac{\omega_{2\,max} + \omega_{2\,min}}{2}$ as the variation of speed is not linear throughout the rotation of the driven shaft.)

Angular Acceleration of Driven Shaft

Differentiating Eq. (6.5) with respect to time (ω_1 = constant)

$$\frac{d\omega_2}{dt} = \omega_1 \frac{d}{dt}\left(\frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta}\right)$$

or

$$\text{acceleration} = \omega_1 \cdot \frac{d\theta}{dt}\frac{d}{d\theta}\left(\frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta}\right)$$

$$= \omega_1^2 \cos\alpha \frac{d}{d\theta}(1-\sin^2\alpha\cos^2\theta)^{-1}$$

$$= \omega_1^2 \cos\alpha\,(-1)\,(1-\sin^2\alpha\cos^2\theta)^{-2}$$

$$(-\sin^2\alpha)\,(2\cos\theta)\,(-\sin\theta)$$

$$= \frac{-\omega_1^2 \cos\alpha \sin^2\alpha \sin 2\theta}{(1-\sin^2\alpha\cos^2\theta)^2} \quad (6.11)$$

This is maximum or minimum when $\frac{d\,(acc)}{d\theta} = 0$. The resulting expression being very cumbersome, the result can be approximated to

$$\cos 2\theta \approx \frac{2\sin^2\alpha}{2-\sin^2\alpha} \quad (6.12)$$

This gives the maximum acceleration of the driven shaft corresponding to two values of θ in the second and the fourth quadrants whereas the minimum acceleration (maximum retardation) corresponds to θ values in the first and the third quadrants.

Care is to be taken to keep the angle between the two shafts to the minimum possible and not to attach excessive masses to the driven shaft. Otherwise, very high alternating stresses due to the angular acceleration and retardation will be set up in the parts of the joint, which are undesirable.

Example 6.8 *Determine the maximum permissible angle between the shaft axes of a universal joint if the driving shaft rotates at 800 rpm and the total fluctuation of speed does not exceed 60 rpm. Also, find the maximum and the minimum speeds of the driven shaft.*

Solution:

$N_1 = 800$ rpm $\qquad \omega_{2\,max} - \omega_{2\,min} = 60$ rpm

We have

Maximum variation,

$$\frac{\omega_{2\,max} - \omega_{2\,min}}{\omega_{mean}} = \frac{1-\cos^2\alpha}{\cos\alpha}$$

or $$\frac{60}{800} = \frac{1-\cos^2\alpha}{\cos\alpha}$$

or $1 - \cos^2\alpha - 0.075\cos\alpha = 0$

$\cos^2\alpha + 0.075\cos\alpha = 1$

$(\cos\alpha + 0.0375)^2 = 1 + (0.0375)^2$

$= 1.0014 = (1.000\ 703)^2$

or

$\cos\alpha = 1.000\ 703 - 0.0375 = 0.963\ 203$

$\alpha = \underline{15.6^\circ}$

Maximum speed of driven shaft

$$= \frac{N_1}{\cos\alpha} = \frac{800}{0.963\ 203}$$

$= \underline{830.6 \text{ rpm}}$

Minimum speed of driven shaft $= N_1 \cos\alpha = 800 \times 0.963\ 203 = \underline{770.6 \text{ rpm}}$

Example 6.9 *The driving shaft of a Hooke's joint rotates at a uniform speed of 400 rpm. If the maximum variation in speed of the driven shaft is ± 5% of the mean speed, determine the greatest permissible angle between the axes of the shafts. What are the maximum and the minimum speeds of the driven shaft?*

Solution:

$N_1 = 400$ rpm

Maximum variation in speed = 0.1 ...(0.05 + 0.05 = 0.1)

or

$$\frac{1-\cos^2\alpha}{\cos\alpha} = 0.1$$

$1 - \cos^2\alpha - 0.1\cos\alpha = 0$

$\cos^2\alpha + 0.1\cos\alpha = 1$

$(\cos\alpha + 0.05)^2 = 1 + (0.05)^2 = 1.0025$

$= (1.001\ 25)^2$

or

$\cos\alpha = 1.001\ 25 - 0.05 = 0.951\ 25$

$\alpha = 17.96^\circ$ or $17^\circ\ 58'$

Minimum speed of driven shaft

$$= \frac{N_1}{\cos\alpha} = \frac{400}{0.95\ 125}$$

$= \underline{420.5 \text{ rpm}}$

Minimum speed of driven shaft

$= N_1 \cos\alpha = 400 \times 0.951\ 25 = \underline{380.5 \text{ rpm}}$

Example 6.10 *A Hooke's joint connects two shafts whose axes intersect at 25°. What will be the angle turned by the driving shaft when the*

(i) velocity ratio is maximum, minimum and unity?

(ii) acceleration of the driven shaft is maximum, minimum (negative) and zero?

Solution:

(i) (a) ω_2/ω_1 is maximum at $\theta = 0^\circ$ and 180°

(b) ω_2/ω_1 is minimum at $\theta = 90^\circ$ and 270°

(c) ω_2/ω_1 is unity when

$\tan\theta = \pm\sqrt{\cos\alpha} = \pm\sqrt{\cos 25^\circ}$

$= \pm\ 0.952$

or $\theta = 43^\circ 35',\ 136^\circ 25',\ 223^\circ 35'$, and $316^\circ 25'$

(ii) Acceleration of driven shaft is maximum or minimum when

$$\cos 2\theta \approx \frac{2\sin^2\alpha}{2-\sin^2\alpha} \approx \frac{2\sin^2 25^\circ}{2-\sin^2 25^\circ} \approx 0.196$$

or $2\theta \approx 78°42'$, $(360° - 78°42')$, $(360° + 78°42')$, $(720° - 78°42')$

or $\theta \approx 39°21'$, $140°39'$, $219°21'$ and $320°39'$

Now,

$$\text{acceleration} = \frac{-\omega_1^2 \cos\alpha \sin^2\alpha \sin\theta}{(1 - \sin^2\alpha \cos^2\theta)^2}$$

Thus, acceleration is positive when $\sin 2\theta$ is negative and is negative when $\sin 2\theta$ is positive.

Corresponding to four values of 2θ found above, $\sin 2\theta$ will be +ve, –ve, +ve and –ve respectively.

Maximum acceleration will be at $140°39'$ and $320°39'$ and minimum acceleration (–ve) will be at $39°21'$ and $219°21'$.

Acceleration is zero when ω_2/ω_1 is maximum or minimum, i.e., at 0°, 90°, 180° and 270°.

Or acceleration is zero when 2θ is zero or when 2θ is 0°, 180°, 360°, 540° or when θ is 0°, 90°, 180° and 270°.

Example 6.11 *The angle between the axes of two shafts joined by Hooke's joint is 25°. The driving shaft rotates at a uniform speed of 180 rpm. The driven shaft carries a steady load of 7.5 kW. Calculate the mass of the flywheel of the driven shaft if its radius of gyration is 150 mm and the output torque of the driven shaft does not vary by more than 15% of the input shaft.*

Solution:

$\alpha = 25°$ $\quad N_1 = 180$ rpm

$P = 7.5$ kW $\quad \omega_1 = \dfrac{2\pi \times 180}{60} = 6\pi$

$k = 0.15$ m $\quad \Delta T = 15\%$

Maximum torque on the driven shaft will be when the acceleration is maximum, i.e., when

$$\cos 2\theta \approx \frac{2\sin^2\alpha}{2 - \sin^2\alpha} \approx \frac{2\sin^2 25°}{2 - \sin^2 25°} \approx 0.196$$

or

$2\theta = 78°42'$ or $281°18'$

$\therefore$ Maximum acceleration

$$= \frac{-\omega_1^2 \cos\alpha \sin^2\alpha \sin 2\theta}{(1 - \sin^2\alpha \cos^2\theta)^2}$$

$$= \frac{-(6\pi)^2 \cos 25° \sin^2 25° \sin 281°18'}{(1 - \sin^2 25° \cos^2 140°39')^2}$$

$= 70.677$ rad/s^2

$P = T\omega_1$

$7500 = T \times 6\pi$

Input torque, $T = 397.9$ N.m

Permissible variation = torque due to acceleration of driven shaft

$397.9 \times 0.15 = I\alpha = mk^2\,\alpha$

or $\quad 397.9 \times 0.15 = m \times (0.15)^2 \times 70.677$

$m = \underline{37.53 \text{ kg}}$

Example 6.12 *A Hooke's joint connects two shafts whose axes intersect at 18°. The driving shaft rotates at a uniform speed of 210 rpm. The driven shaft with attached masses has a mass of 60 kg and radius of gyration of 120 mm. Determine the*

(i) torque required at the driving shaft if a steady torque of 180 N.m resists rotation of the driven shaft and the angle of rotation is 45°

(ii) angle between the shafts at which the total fluctuation of speed of the driven shaft is limited to 18 rpm

Solution:

$m = 60$ kg $\quad k = 120$ mm $\quad N = 210$ rpm

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 210}{60} = 22 \text{ rad/s}$$

Maximum acceleration

$$= \frac{-\omega_1^2 \cos\alpha \sin^2\alpha \sin 2\theta}{(1 - \sin^2\alpha \cos^2\theta)^2}$$

$$= \frac{-(22)^2 \cos 18° \sin^2 18° \sin 90°}{(1 - \sin^2 18° \cos^2 45°)^2}$$

$$= -\frac{43.956}{0.907}$$

$= -48.47$ rad/s^2

The negative sign indicates that it is retardation at the instant.

Torque required for retardation of the driven shaft = $I\,\alpha = mk^2.\alpha$

$= 60 \times 0.12^2 \times (-48.47)$

$= -41.88$ N.m

Total torque required on the driven shaft, T_2

= Steady torque + Accelerating torque

= 180 + (– 41.88)

= 138.12 N.m

Now as $P = T_1\omega_1 = T_2\omega_2$

$$\therefore\ T_1 = T_2\frac{\omega_2}{\omega_1} = T_2\frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta}$$

$$= 138.12\times\frac{\cos 18^\circ}{1-\sin^2 18^\circ\cos^2 45^\circ}$$

$= \underline{137.99\text{ N.m}}$

$$\text{Maximum variation} = \frac{1-\cos^2\alpha}{\cos\alpha}$$

or $$\frac{\omega_{2\,max}-\omega_{2\,min}}{\omega_{mean}} = \frac{1-\cos^2\alpha}{\cos\alpha}$$

or $$\frac{18}{180} = \frac{1-\cos^2\alpha}{\cos\alpha}$$

or $1-\cos^2\alpha - 0.1\cos\alpha = 0$

$\cos^2\alpha + 0.1\cos\alpha = 1$

On solving,

$\alpha = \underline{17.96^\circ}$

6.7 DOUBLE HOOKE'S JOINT

In a single Hooke's joint, the speed of the driven shaft is not uniform although the driving shaft rotates at a uniform speed. To get uniform velocity ratio, a double Hooke's joint has to be used. In a double Hooke's joint, two universal joints and an intermediate shaft are used. If the angular misalignment between each shaft and the intermediate shaft is equal, the driving and the driven shafts remain in exact angular alignment, though the intermediate shaft rotates with varying speed.

A single Hooke's joint was analysed assuming the axes of the two shafts and the fork of the driving shaft to be horizontal. The results showed that the speed of the driven shaft is the same after an angular displacement of 180°. Therefore, it is immaterial whether the driven shaft makes the angle α with the axis of the driving shaft to its left or right.

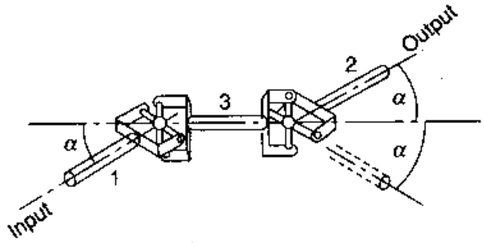

Fig. 6.29

Thus, to have a constant velocity ratio

- the driving and the driven shafts should make equal angles with the intermediate shaft, and
- the forks of the intermediate shaft should lie in the same plane.

Let γ be the angle turned by the intermediate shaft 3 while the angle turned by the driving shaft 1 and the driven shaft 2 be θ and φ respectively as before (Fig. 6.29).

Then,

$\tan\theta = \cos\alpha\tan\gamma$ (fork of shaft 1 horizontal)

and

$\tan\varphi = \cos\alpha\tan\gamma$ (fork of shaft 2 horizontal)

$\therefore$ $\theta = \varphi$

This type of joint can be used for two intersecting shafts as well as for two parallel shafts.

However, if somehow the forks of the intermediate shafts lie in planes perpendicular to each other, the variation of speed of the driven shaft will be there.

$$\left(\frac{\omega_3}{\omega_1}\right)_{min} = \cos\alpha \qquad \text{(fork of the shaft 1 horizontal)}$$

$$\left(\frac{\omega_2}{\omega_3}\right)_{min} = \cos\alpha \qquad \text{(fork of the shaft 1 horizontal)}$$

$$\therefore \qquad \left(\frac{\omega_2}{\omega_1}\right)_{min} = \cos^2\alpha \qquad (6.13)$$

Similarly,

$$\left(\frac{\omega_2}{\omega_1}\right)_{max} = \frac{1}{\cos^2\alpha} \qquad (6.14)$$

Therefore, the maximum variation (fluctuation) of speed of the driven shaft is from $\cos^2\alpha$ to $1/\cos^2\alpha$.

Example 6.13 *The driving shaft of a double Hooke's joint rotates at 400 rpm. The angle of the driving and of the driven shaft with the intermediate shaft is 20°. If somehow the forks of the intermediate shaft lie in planes perpendicular to each other, determine the maximum and the minimum velocities of the driven shaft.*

Solution:

$$\omega_{2min} = \omega_1 \cos^2\alpha$$

or $$N_{2min} = N_1\cos^2\alpha = 400 \times \cos^2 20°$$

$$= \underline{353.2 \text{ rpm}}$$

$$N_{2max} = \frac{N_1}{\cos^2\alpha} = \frac{400}{\cos^2 20°}$$

$$= \underline{453 \text{ rpm}}$$

Summary

1. An exact straight-line mechanism guides a reciprocating part in an exact straight line.
2. An approximate straight-line mechanism is designed in such a way that the middle and the two extreme positions of the guided point are in a straight line and the intermediate positions deviate as little as possible from the line.
3. A *pantograph* is a four-bar linkage used to produce paths exactly similar to the ones traced out by a point on the linkage. The paths so produced are, usually, on an enlarged or a reduced scale.
4. An *indicator* of a reciprocating engine is an instrument that keeps the graphical record of pressure inside the cylinder during the piston stroke.
5. The fundamental equation of correct gearing for automobiles is, $\cot\varphi - \cot\theta = \frac{w}{l}$. Mechanisms that fulfill this fundamental equation are known as *steering gears*.
6. Two main types of steering gears are Davis steering gear, and Ackermann steering gear.
7. A *Davis steering gear* has sliding pairs which means more friction and easy wearing. The gear fulfils the fundamental equation of gearing in all the positions.
8. An *Ackermann steering gear* has only turning pairs and thus is preferred. Its drawback is that it fulfils the fundamental equation of correct gearing at the middle and the two extreme positions and not in all positions.
9. A *Hooke's joint* commonly known as a *universal joint*, is used to connect two non-parallel and intersecting shafts.
10. Speed of the driven shaft is minimum when θ = 90° or 270°, and the minimum speed is given by $\omega_2 = \omega_1\cos\alpha$.
11. Speed of the driven shaft is maximum when θ = 0° or 180°, and the maximum speed is given by $\omega_2 = \omega_1/\cos\alpha$.

Exercises

1. What is a pantograph? Show that it can produce paths exactly similar to the ones traced out by a point on a link on an enlarged or a reduced scale.
2. Enumerate straight-line mechanisms. Why are they classified into exact and approximate straight-line mechanisms?
3. Sketch a Paucellier mechanism. Show that it can be used to trace a straight line.
4. Prove that a point on one of links of a Hart mechanism traces a straight line on the movement of its links.
5. What is a Scott–Russel mechanism? What is its limitation? How is it modified?
6. In what way is a Grass–Hopper mechanism a derivation of the modified Scott–Russel mechanism?
7. How can you show that a Watt mechanism traces an approximate straight line?
8. How can we ensure that a Tchebicheff mechanism traces an approximate straight line?
9. Prove that a Kempe's mechanism traces an exact straight line using two identical mechanisms.
10. Discuss some of the applications of parallel linkages.
11. What is an engine indicator? Describe any one of them.
12. With the help of neat sketch discuss the working of a Crosby indicator.
13. Describe the function of a Thomson or a Dobbie McInnes Indicator.
14. What is an automobile steering gear? What are its types? Which steering gear is preferred and why?
15. What is fundamental equation of steering gears? Which steering gear fulfils this condition?
16. An Ackermann steering gear does not satisfy the fundamental equation of a steering gear at all positions. Yet it is widely used. Why?
17. What is a Hooke's joint? Where is it used?
18. Derive an expression for the ratio of angular velocities of the shafts of a Hooke's joint.
19. Sketch a polar velocity diagram of a Hooke's joint and mark its salient features.
20. Design and dimension a pantograph to be used to double the size of a pattern.
    (In Fig. 6.1, make $\frac{OC}{OD} = \frac{OR}{OP} = 2$. *Drawing tool at R; P traces the pattern*)
21. Design and dimension a pantograph which will decrease pattern dimensions by 30%.
    (In Fig. 6.1, make
    $\frac{OC}{OD} = \frac{OR}{OP} = \frac{100}{100-30}$; *Drawing tool at R; P traces the pattern*)
22. Design and dimension a pantograph that can be used to decrease pattern dimensions by 15%. The fixed pivot should lie between the tracing point and the marking point (tool holder).
    (In Fig. 6.1, make $\frac{CD}{OD} = \frac{RP}{OP} = \frac{100}{100-15}$;
    P, the fixed pivot; Drawing tool at O; R traces the pattern)
23. In Fig. 6.30, the dimensions of the various links are such that

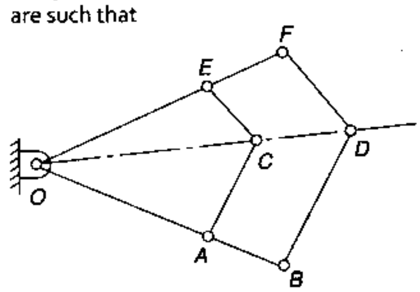

Fig. 6.30

$$\frac{OA}{OB} = \frac{OE}{OF} = \frac{AC}{BD} = \frac{EC}{FD}$$

Show that if C traces any path then D will describe a similar path and vice-versa.

24. Figure 6.31 shows a straight-line Watt mechanism. Plot the path of point P and mark and measure the straight line segment of the path of P.

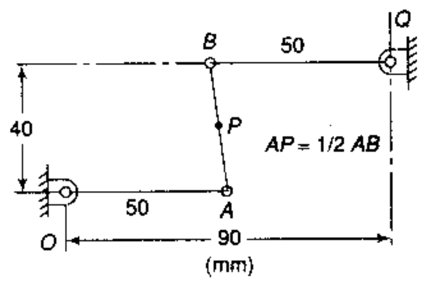

Fig. 6.31

25. Figure 6.32 shows a Robert straight-line mechanism in which *ABCD* is a four-bar linkage. The cranks *AB* and *DC* are equal and the connecting rod *BC* is one-half as long as the line of centres *AD*. *P* is a point rigidly attached to the connecting rod and lying on the midpoint of *AD* when *BC* is parallel to *AD*. Show that the point *P* moves in an approximately straight line for small displacement of the cranks.
(Note: *For better results take AB or DC > 0.6 AD*)

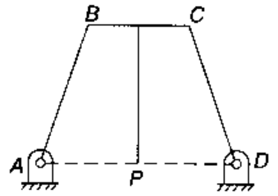

Fig. 6.32

26. In the Robert mechanism (Fig. 6.32) if $AB = BC = CD = AD/2$, locate the point *P* on the central vertical arm that approximately describes a straight line. (At a length 1.3 *BC* below *BC*)
27. In a Watt parallel motion (Fig. 6.10), the links *OA* and *QB* are perpendicular to the link *AB* in the mean position. The lengths of the moving links are $OA = 120$ mm, $QB = 200$ mm and $AB = 175$ mm.
Locate the position of a point *P* on *AB* to trace approximately a straight line motion. Also, trace the locus of *P* for all possible movements. ($AP = 109.3$ mm)
28. In a Watt mechanism of the type shown in Fig. 6.33, the links *OA* and *QB* are perpendicular to the link *AB* in the mean position. If $OA = 45$ mm, $QB = 90$ mm and $AB = 60$ mm, find the point *P* on the link *AB* produced for approximate straight-line motion of point *P*.

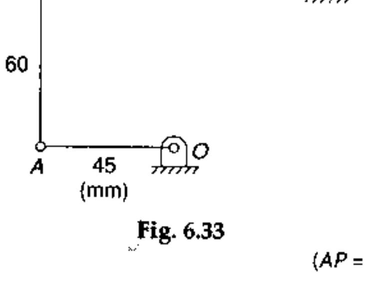

Fig. 6.33

($AP = 120$ mm)

29. In a Davis steering gear, the length of the car between axles is 2.4 m, and the steering pivots are 1.35 m apart. Determine the inclination of the track arms to the longitudinal axis of the car when the car moves in a straight path.
(15°42′)
30. In a Hooke's joint, the angle between the two shafts is 15°. Find the angles turned by the driving shaft when the velocity of the driven shaft is maximum, minimum and equal to that of the driving shaft. Also, determine when the driven shaft will have the maximum acceleration and retardation.
(Max. vel. at 0° and 180°; min. at 90° and 270°; equal to 44°30′. 135°30′, 224°30′ and 315°30′; Max. acc. at 137° and 317°; and Max. ret. at 43° and 223°)
31. The driving shaft of a Hooke's joint has a uniform angular speed of 280 rpm. Determine the maximum permissible angle between the axes of the shafts to permit a maximum variation in speed of the driven shaft by 8% of the mean speed.
(22.6°)
32. The two shafts of a Hooke's coupling have their axes inclined at 20°. The shaft *A* revolves at a uniform speed of 1000 rpm. The shaft *B* carries a flywheel of mass 30 kg. If the radius of gyration of the flywheel is 100 mm, find the maximum torque in shaft *B*.
(411 N.m)
33. In a double universal coupling joining two shafts, the intermediate shaft is inclined at 10° to each. The input and the output forks on the intermediate shaft have been assembled inadvertently at 90° to one another. Determine the maximum and the least velocities of the output shaft if the speed of the input shaft is 500 rpm. Also, find the coefficient of fluctuation in speed.
(515.5 rpm; 484.9 rpm; 0.06)

7 CAMS

Introduction

A cam is a mechanical member used to impart desired motion to a follower by direct contact. The cam may be rotating or reciprocating whereas the follower may be rotating, reciprocating or oscillating. Complicated output motions which are otherwise difficult to achieve can easily be produced with the help of cams. Cams are widely used in automatic machines, internal combustion engines, machine tools, printing control mechanisms, and so on. They are manufactured usually by die-casting, milling or by punch-presses.

A cam and the follower combination belong to the category of higher pairs. Necessary elements of a cam mechanism are

- A driver member known as the *cam*
- A driven member called the *follower*
- A *frame* which supports the cam and guides the follower

The chapter outlines the methods of drawing the cam profiles as well as analysis to determine the velocity and acceleration at various positions of the follower. This information is useful in determining the smooth operation of the cam.

7.1 TYPES OF CAMS

Cams are classified according to

1. shape,
2. follower movement, and
3. manner of constraint of the follower.

According to Shape

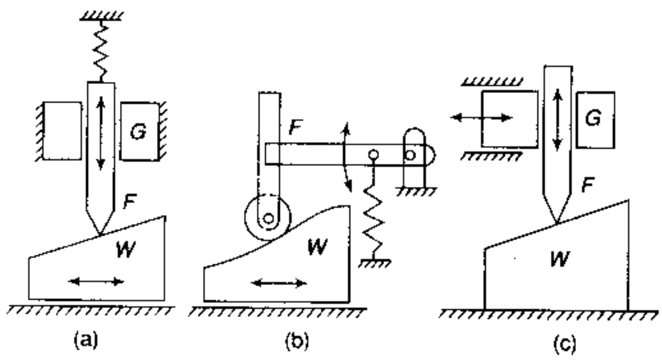

Fig. 7.1

1. ***Wedge and Flat Cams*** A wedge cam has a wedge W which, in general, has a translational motion [Figs 7.1(a) and (b). The follower F can either translate [Fig. 7.1(a)] or oscillate [Fig. 7.1(b)]. A spring is, usually, used to maintain the contact between the cam and

the follower. In Fig. 7.1(c), the cam is stationary and the follower constraint or guide G causes the relative motion of the cam and the follower.

In stead of using a wedge, a flat plate with a groove can also be used. In the groove the follower is held as shown in Fig. 7.2. Thus, a positive drive is achieved without the use of a spring.

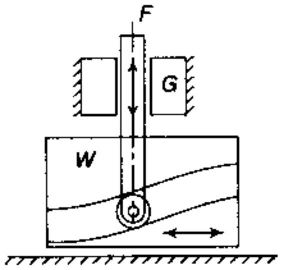

Fig. 7.2

2. ***Radial or Disc Cams*** A cam in which the follower moves radially from the centre of rotation of the cam is known as a radial or a disc cam [Fig. 7.3(a) and (b)]. Radial cams are very popular due to their simplicity and compactness.

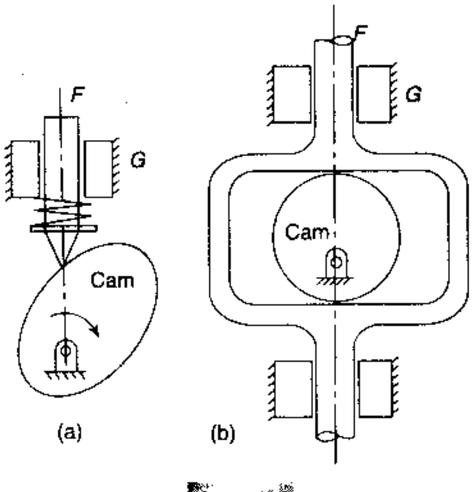

Fig. 7.3

3. ***Spiral Cams*** A spiral cam is a face cam in which a groove is cut in the form of a spiral as shown in Fig. 7.4. The spiral groove consists of teeth which mesh with a pin gear follower. The velocity of the follower is proportional to the radial distance of the groove from the axis of the cam.

   The use of such a cam is limited as the cam has to reverse the direction to reset the position of the follower. It finds its use in computers.

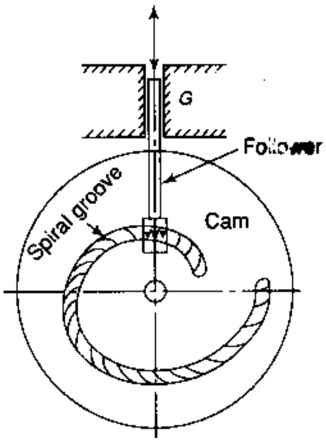

Fig. 7.4

4. ***Cylindrical Cams*** In a cylindrical cam, a cylinder which has a circumferential contour cut in the surface, rotates about its axis. The follower motion can be of two types as follows:

   In the first type, a groove is cut on the surface of the cam and a roller follower has a constrained (or positive) oscillating motion [Fig. 7.5(a)]. Another type is an end cam in which the end of the cylinder is the working surface (7.5b). A spring-loaded follower translates along or parallel to the axis of the rotating cylinder.

Cylindrical cams are also known as *barrel* or *drum* cams.

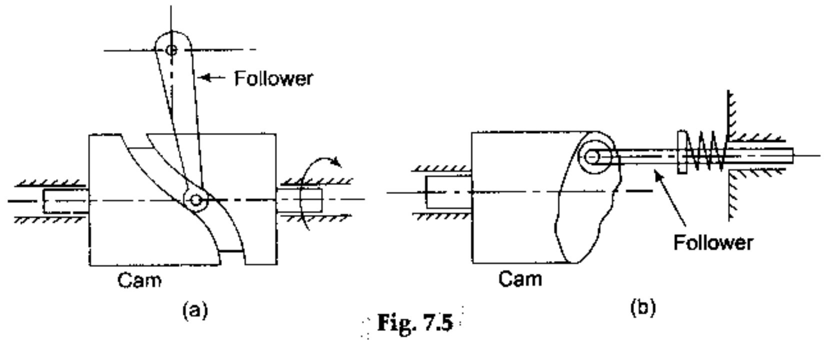

Fig. 7.5

5. ***Conjugate Cams*** A conjugate cam is a double-disc cam, the two discs being keyed together and are in constant touch with the two rollers of a follower (Fig. 7.6). Thus, the follower has a positive constraint. Such a type of cam is preferred when the requirements are low wear, low noise, better control of the follower, high speed, high dynamic loads, etc.

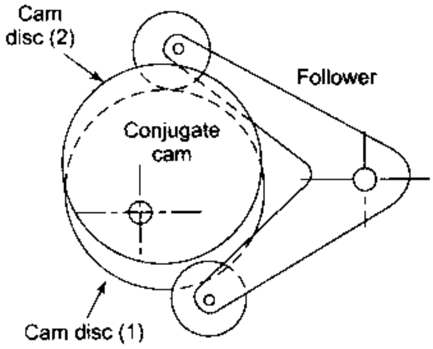

Fig. 7.6

6. ***Globoidal Cams*** A globoidal cam can have two types of surfaces, convex or concave. A circumferential contour is cut on the surface of rotation of the cam to impart motion to the follower which has an oscillatory motion (Fig. 7.7). The application of such cams is limited to moderate speeds and where the angle of oscillation of the follower is large.

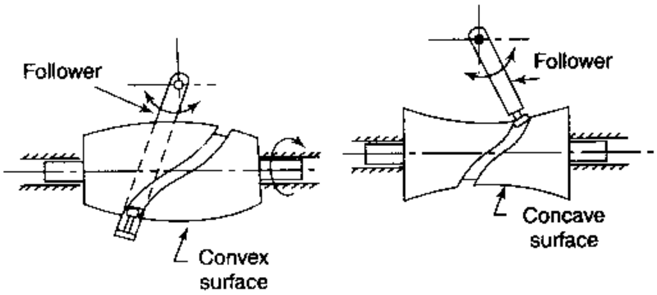

Fig. 7.7

7. ***Spherical Cams*** In a spherical cam, the follower oscillates about an axis perpendicular to the axis of rotation of the cam. Note that in a disc cam, the follower oscillates about an axis parallel to the axis of rotation of the cam.

A spherical cam is in the form of a spherical surface which transmits motion to the follower (Fig. 7.8).

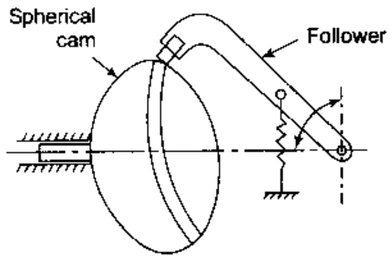

Fig. 7.8

According to Follower Movement

The motions of the followers are distinguished from each other by the dwells they have. A dwell is the zero displacement or the absence of motion of the follower during the motion of the cam.

Cams are classified according to the motions of the followers in the following ways:

1. ***Rise-Return-Rise* (R-R-R)** In this, there is alternate rise and return of the follower with no periods of dwells (Fig. 7.9a). Its use is very limited in the industry. The follower has a linear or an angular displacement.
2. ***Dwell-Rise-Return-Dwell* (D-R-R-D)** In such a type of cam, there is rise and return of the follower after a dwell [Fig. 7.9(b)]. This type is used more frequently than the R-R-R type of cam.
3. ***Dwell-Rise-Dwell-Return-Dwell* (D-R-D-R-D)** It is the most widely used type of cam. The dwelling of the cam is followed by rise and dwell and subsequently by return and dwell as shown in Fig. 7.9(c). In case the return of the follower is by a fall [Fig. 7.9(d)], the motion may be known as Dwell-Rise-Dwell (D-R-D).

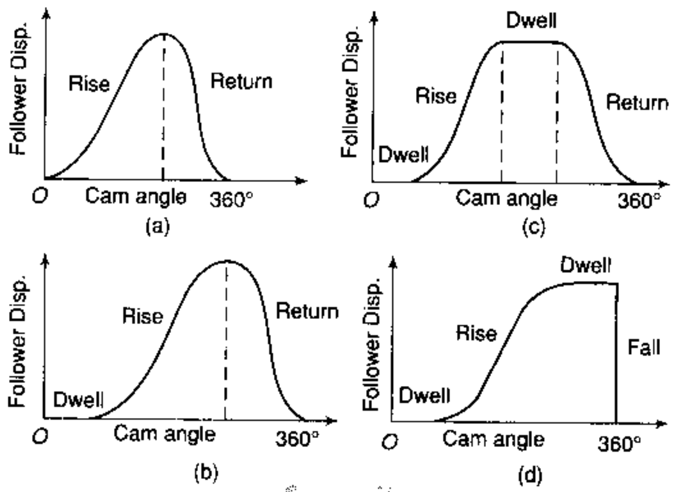

Fig. 7.9

According to Manner of Constraint of the Follower

To reproduce exactly the motion transmitted by the cam to the follower, it is necessary that the two remain in touch at all speeds and at all times. The cams can be classified according to the manner in which this is achieved.

1. ***Pre-loaded Spring Cam*** A pre-loaded compression spring is used for the purpose of keeping the contact between the cam and the follower [Figs 7.1(a) and (b), 7.3(a), 7.5(b) and 7.8].
2. ***Positive-drive Cam*** In this type, constant touch between the cam and the follower is maintained by a roller follower operating in the groove of a cam [Figs 7.2, 7.3(b). 7.4, 7.5(a) and 7.7]. The follower cannot go out of this groove under the normal working operations. A constrained or positive drive is also obtained by the use of a conjugate cam (Fig. 7.6).
3. ***Gravity Cam*** If the rise of the cam is achieved by the rising surface of the cam and the return by the force of gravity or due to the weight of the cam, the cam is known as a gravity cam. Figure 7.2(c) shows such a cam. However, these cams are not preferred due to their uncertain behaviour.

7.2 TYPES OF FOLLOWERS

Cam followers are classified according to the
1. shape,
2. movement, and
3. location of line of movement.

According to Shape

1. ***Knife-edge Follower*** It is quite simple in construction. Figure 7.1(a) shows such a follower. However, its use is limited as it produces a great wear of the surface at the point of contact.
2. ***Roller Follower*** It is a widely used cam follower and has a cylindrical roller free to rotate about a pin joint [Figs 7.1(b), 7.2, 7.5, 7.8]. At low speeds, the follower has a pure rolling action, but at high speeds, some sliding also occurs.

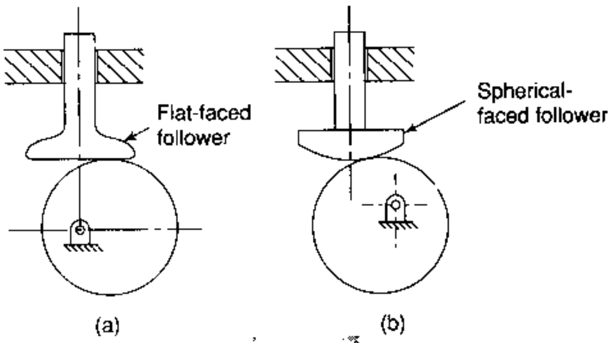

Fig. 7.10

In case of steep rise, a roller follower jams the cam and, therefore, is not preferred.

3. ***Mushroom Follower*** A mushroom follower (Fig. 7.10) has the advantage that it does not pose the problem of jamming the cam. However, high surface stresses and wear are quite high due to deflection and misalignment if a flat-faced follower is used [Fig. 7.10(a)]. These disadvantages are reduced if a spherical-faced follower [Fig. 7.10(b)] is used instead of a flat-faced follower.

According to Movement

1. ***Reciprocating Follower*** In this type, as the cam rotates, the follower reciprocates or translates in the guides [Fig. 7.1(a)].
2. ***Oscillating Follower*** The follower is pivoted at a suitable point on the frame and oscillates as the cam makes the rotary motion [Fig. 7.1(b)].

A cam with two oscillating followers. This cam operates the inlet as well as the exhaust valves of the engine

According to Location of Line of Movement

1. ***Radial Follower*** The follower is known as a radial follower if the line of movement of the follower passes through the centre of rotation of the cam [Figs 7.3(a) and (b)].
2. ***Offset Follower*** If the line of movement of the roller follower is offset from the centre of rotation of the cam, the follower is known as an offset follower [Fig. 7.10(b)].

7.3 DEFINITIONS

With reference to Fig. 7.11, some definitions are given below:

Base Circle It is the smallest circle tangent to the cam profile (contour) drawn from the centre of rotation of a radial cam.

Trace point It is a reference point on the follower to trace the cam profile such as the knife-edge of a knife-edged follower and centre of the roller of a roller follower.

Pitch Curve It is the curve drawn by the trace point assuming that the cam is fixed, and the trace point of the follower rotates around the cam.

Pressure Angle The pressure angle, representing the steepness of the cam profile, is the angle between the normal to the pitch curve at

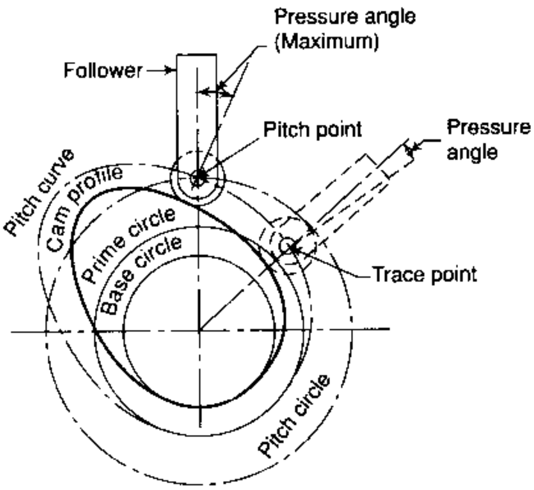

Fig. 7.11

a point and the direction of the follower motion. It varies in magnitude at all instants of the follower motion. A high value of the maximum pressure angle is not desired as it might jam the follower in the bearings.

Pitch Point It is the point on the pitch curve at which the pressure angle is maximum.

Pitch Circle It is the circle passing through the pitch point and concentric with the base circle.

Prime Circle The smallest circle drawn tangent to the pitch curve is known as the prime circle.

7.4 FOLLOWER DISPLACEMENT PROGRAMMING

As a cam rotates about the axis, it imparts a specific motion to the follower which is repeated with each revolution of the cam. Thus, it is enough to know the motion of the follower for only one revolution. During rotation of the cam through one revolution, the follower is made to execute a series of events such as rises, dwells and returns. The motion of the cam can be represented on a graph, the x-axis of which may represent the angular displacement of the cam and y-axis, the angular or the linear displacement of the follower. The follower displacement is measured from its lowest position and is plotted with the same scale as is to be used in the layout of the cam profile. The following terms are used with reference to the angular motion of the cam:

- ***Angle of Ascent*** **(ϕ_a)** It is the angle through which the cam turns during the time the follower rises.
- ***Angle of Dwell*** **(δ)** The angle of dwell is the angle through which the cam turns while the follower remains stationary at the highest or the lowest position.
- ***Angle of Descent*** **(ϕ_d)** It is the angle through which the cam turns during the time the follower returns to the initial position.
- ***Angle of Action*** The angle of action is the total angle moved by the cam during the time, between the beginning of rise and the end of the return of the follower.

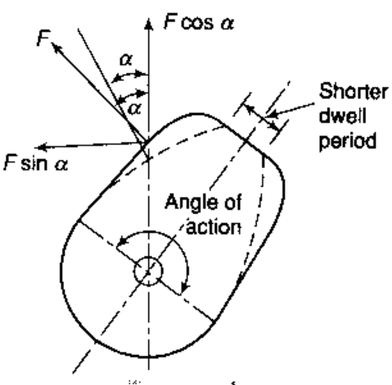

Fig. 7.12

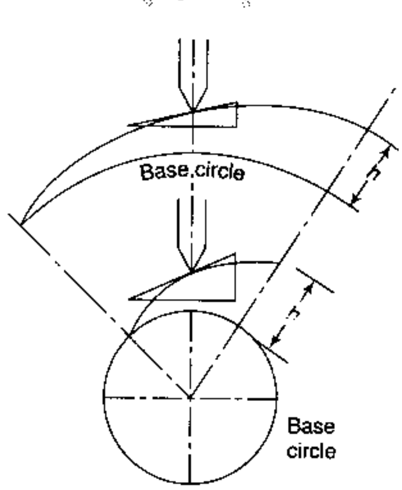

Fig. 7.13

To satisfy the given requirements of the follower displacement, a programme can be made keeping in view the following points:

1. In a given specific interval of time, due consideration to the velocity and the acceleration must be given, the effects of which are manifested as inertia loads. The dynamic effects of acceleration, usually limit the speed of the cams. Moreover, effects of jerks (rate of change of acceleration) in case of high-speed mechanisms produce vibrations of the system, which is undesirable for a follower motion. Though it is very difficult to completely eliminate jerk, efforts are to be made to keep it within tolerable limits.
2. The force exerted by a cam on the follower is always normal to the surface of the cam at the point of contact. The vertical component ($F \cos\alpha$) lifts the follower whereas the horizontal component ($F \sin\alpha$) exerts lateral pressure on the bearing as shown in Fig. 7.12.

   In order to reduce the lateral pressure or $F \sin \alpha$, α has to be decreased which means making the surface more convex and longer (dotted lines). This results in reduced velocity of the follower and more time for the same rise. This also reduces the dwell period for a fixed angle of action. In internal combustion engines, a shorter dwell period means a smaller period of valve-opening, resulting in less fuel per cycle and lesser power production. Thus, the minimum value of α cannot be reduced from a certain value.

3. The size of the base circle controls the pressure angle. As shown in Fig. 7.13, the increase in the base circle diameter increases the length of the arc of the circle upon which the wedge (the raised portion) is to be made. A short wedge for a given rise requires a steep rise or a higher pressure angle, thus increasing the lateral force.

7.5 DERIVATIVES OF FOLLOWER MOTION

The derivatives of follower motion can be *kinematic* (with respect to θ) which relate to the geometry of the cam system or *physical* (with respect to time) which relate to the motion of the follower of the cam system.

Kinematic Derivatives

A displacement diagram of the follower motion is plotted with the cam angle θ as the abscissa and the follower linear or angular motion as the ordinate. Thus, it is a graph that relates the input and the output of the cam system. Mathematically, if s is the displacement of the follower. Then

$$s = s(\theta)$$

Differentiating it with respect to θ provides the first derivative,

$$\dot{s}(\theta) = \frac{ds}{d\theta}$$

It represents the slope or the steepness of the displacement curve at each position of the cam angle. A higher value of this means a steep rise or fall which hampers the smooth running of the cam.

The second derivative is represented by

$$\ddot{s}(\theta) = \frac{d^2s}{d\theta^2}$$

This derivative is related to the radius of curvature of the cam at different points along its profile and is in inverse proportion. Thus, with an increase in its value, the radius of curvature decreases. If its value becomes infinity, the cam profile becomes pointed at that position which is undesirable from the point of view of stresses between the cam and follower surfaces.

The next derivative can also be taken if desired:

$$\dddot{s}(\theta) = \frac{d^3s}{d\theta^3}$$

It is not easy to describe it geometrically. However, it should also be controlled as far as possible while choosing the shape of the displacement diagram for smooth working of the cam.

Physical Derivatives

We have, $s = s(\theta)$ and $\theta = \theta(t)$

Taking the first derivative with respect to time,

$$\dot{s} = \frac{ds}{dt} = \frac{ds}{d\theta}\frac{d\theta}{dt} = \omega\frac{ds}{d\theta}$$

which represents the velocity of the follower.

The second derivative is

$$\ddot{s} = \frac{d^2s}{dt^2} = \omega^2 \frac{d^2s}{d\theta^2}$$

It represents the *acceleration* of the follower. A higher value of acceleration means a higher inertia force.

A third derivative is known as the *jerk*.

$$\dddot{s} = \frac{d^3s}{dt^3} = \omega^3 \frac{d^3s}{d\theta^3}$$

For smooth movement of the follower, even the high values of the jerk are undesirable in case of high-speed cams.

7.6 HIGH-SPEED CAMS

A real follower always has some mass and when multiplied by acceleration, inertia force of the follower is obtained. This force is always felt at the contact point of the follower with the cam surface and at the bearings. An acceleration curve with abrupt changes exerts abrupt stresses on the cam surfaces and at the bearings accompanied by detrimental effects such as surface wear and noise. All this may lead to an early failure of the cam system. Thus, it is very important to give due consideration to velocity and acceleration curves while choosing a displacement diagram. They should not have any step changes.

In low-speed applications, cams with discontinuous acceleration characteristics may not show any undesirable characteristic, but at higher speeds such cams are certainly bound to show the same. The higher the speed, the higher is the need for smooth curves. At very high speeds, even the jerk (related to rate of change of acceleration or force) is made continuous as well. For most of the applications, however, this may not be needed. In Section 7.8, standard cam motions have been discussed from which some comparison can easily be made for suitable selection.

7.7 UNDERCUTTING

Sometimes, it may happen that the prime circle of a cam is proportioned to provide a satisfactory pressure angle; still the follower may not be completing the desired motion. This can happen if the curvature of the pitch curve is too sharp.

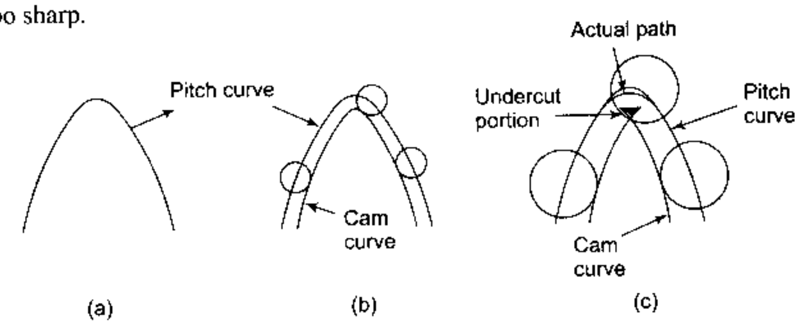

Fig. 7.14

Figure 7.14(a) shows the pitch curve of a cam. In Fig. 7.14(b), a roller follower is shown generating this curve. In Fig. 7.14(c), a larger roller is shown trying to generate this curve. It can easily be observed that the cam curve loops over itself in order to realize the profile of the pitch curve. As it is impossible to produce such a cam profile, the result is that the cam will be undercut and become a pointed cam. Now when the roller follower will be made to move over this cam, it will not be producing the desired motion.

It may be observed that the cam will be pointed if the radius of the roller is equal to the radius of curvature of the pitch curve. Thus, to have a minimum radius of curvature of the cam profile, the radius of curvature of the prime circle must always be greater than that of the radius of the roller.

7.8 MOTIONS OF THE FOLLOWER

Though the follower can be made to have any type of desired motion, knowledge of the existing motion programmes saves time and labour while designing the cams.

Following are some basic displacement programmes:

1. Simple Harmonic Motion (SHM)

This is a popular follower motion and is easy to lay out.

Let s = follower displacement (instantaneous)
h = maximum follower displacement
v = velocity of the follower
f = acceleration of the follower
θ = cam rotation angle (instantaneous)
φ = cam rotation angle for the maximum follower displacement
β = angle on the harmonic circle.

Construction The follower rises through a distance h while the cam turns through an angle φ. Construct the follower displacement curve as follows (Fig. 7.15).

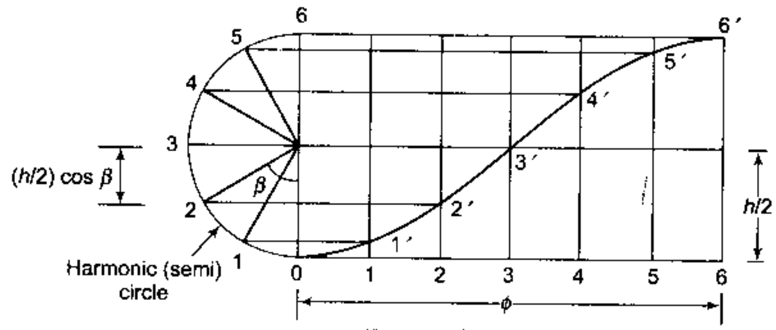

Fig. 7.15

(i) Draw a semicircle with cam rise (or fall) as the diameter. This is, usually known as the harmonic (semi) circle. Divide this semicircle into n equal arcs (n even).

(ii) Divide the cam displacement interval into n equal divisions.

(iii) Project the intercepts of the harmonic semicircle to the corresponding divisions of the cam displacement interval.

(iv) Join the points with a smooth curve to obtain the required harmonic curve.

Displacement At any instant, displacement of the follower is given by,

$$s = \frac{h}{2} - \frac{h}{2}\cos\beta$$

$$= \frac{h}{2}(1 - \cos\beta) \qquad \text{(i)}$$

For the rise (or fall) h of the follower displacement, the cam is rotated through an angle φ whereas a point on the harmonic semicircle traverses an angle π. Thus, the cam rotation is proportional to the angle turned by the point on the harmonic semicircle, i.e.

$$\beta = \pi\frac{\theta}{\varphi}$$

Thus β can be replaced by θ and φ in Eq. (i) above,

$$s = \frac{h}{2}\left(1 - \cos\frac{\pi\theta}{\varphi}\right) \qquad (7.1)$$

The expression is also valid for β more than 90°. In that case, cos β or cos $\pi\theta/\varphi$ becomes negative so that s is again positive and more than $h/2$.

Let ω = Angular velocity of the cam

$\therefore \qquad \theta = \omega t$

and $\qquad s = \frac{h}{2}\left(1 - \cos\frac{\pi\omega t}{\varphi}\right)$

$$v = \frac{ds}{dt} = \frac{h}{2}\frac{\pi\omega}{\varphi}\sin\frac{\pi\omega t}{\varphi}$$

$$= \frac{h}{2}\frac{\pi\omega}{\varphi}\sin\frac{\pi\theta}{\varphi} \qquad (7.2)$$

$$v_{max} = \frac{h}{2}\frac{\pi\omega}{\varphi} \text{ at } \theta = \frac{\varphi}{2} \qquad (7.3)$$

$$f = \frac{dv}{dt} = \frac{h}{2}\left(\frac{\pi\omega}{\varphi}\right)^2\cos\frac{\pi\omega t}{\varphi}$$

$$= \frac{h}{2}\left(\frac{\pi\omega}{\varphi}\right)^2\cos\frac{\pi\theta}{\varphi} \qquad (7.4)$$

$$f_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\varphi}\right)^2 \text{ at } \theta = 0 \qquad (7.5)$$

Let φ_a = angle of ascent

φ_d = angle of descent

$\delta_{1,2}$ = angles of dwells

It can be seen from the plots of Fig. 7.16 that there is an abrupt change of acceleration from zero to maximum at the beginning of the follower motion and also from maximum (negative) to zero at the end of the follower motion when the follower rises. Similar abruption would also be there at the start and end of the return motion. As these abrupt changes result in infinite jerk, vibration and noise, the programme should be adopted only for low or moderate cam speeds.

2. Constant Acceleration and Deceleration (Parabolic)

In such a follower programme, there is acceleration in the first half of the follower motion whereas it is deceleration during the later half. The displacement curve is found to be parabolic in this case. The magnitude of the acceleration and the deceleration is the same and constant in the two halves.

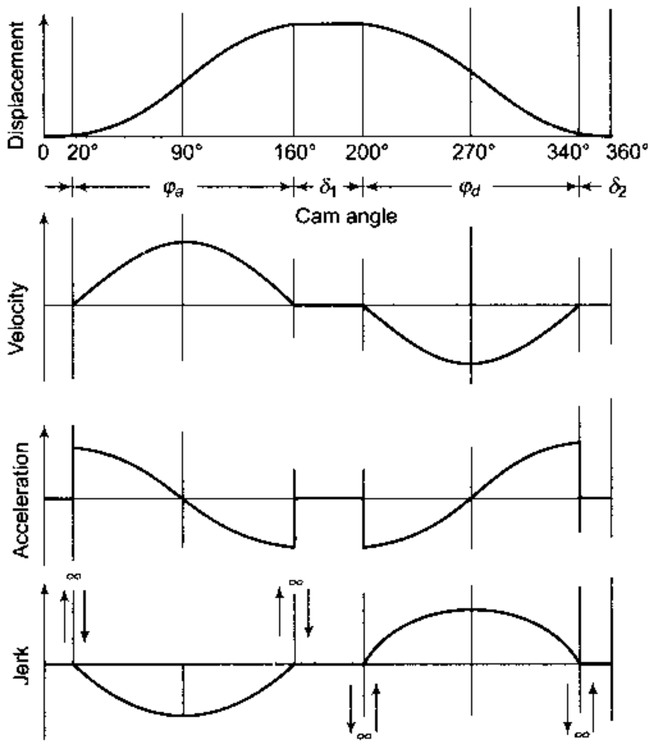

Fig. 7.16

Construction Refer Fig. 7.17(a).

(i) Divide each half of the cam displacement interval into n equal divisions.

(ii) Divide half the follower rise into n^2 equal divisions.

(iii) Project 1^2 displacement interval to the first ordinate of the cam displacement, 2^2 to the second ordinate, 3^2 to the third, and so on.

(iv) The second half of the curve is similar to the first half.

Alternatively, divide half of the follower rise at the central ordinate of the cam displacement into n equal divisions [Fig. 17.17(b)]. Joining the zero point with the first division gives 1^2 on the first ordinate, with the second division 2^2 on the second ordinate, and so on.

The equation for the linear motion with constant acceleration f (during the first half of the follower motion) is found as follows:

$$s = v_0 t + \frac{1}{2} f t^2$$

where v_o is the initial velocity at the start of the motion (rise or fall) and is zero in this case.

$$\therefore \qquad S = \frac{1}{2} f t^2$$

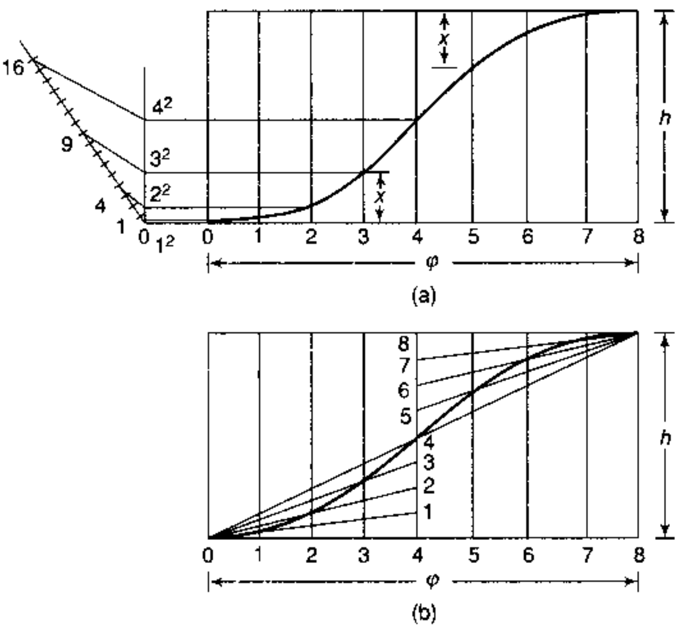

Fig. 7.17

or $$f = \frac{2s}{t^2} = \text{constant} \tag{7.6}$$

As f is constant during the accelerating period, considering the follower at the midway,

$$s = \frac{h}{2} \quad \text{and} \quad t = \frac{\varphi / 2}{\omega}$$

$$\therefore \quad f = \frac{2h/2}{\varphi^2 / 4\omega^2} = \frac{4h\omega^2}{\varphi^2} \tag{7.6a}$$

The velocity is linear during the period and is given by

$$v = \frac{ds}{dt} = \frac{1}{2} \times 2ft = ft \tag{7.7}$$

$$= \frac{4h\omega^2}{\varphi^2} \frac{\theta}{\omega} \qquad (\theta = \omega t)$$

$$= \frac{4h\omega}{\varphi^2}\theta \tag{7.7a}$$

The velocity is maximum when θ is maximum or the follower is at the midway, i.e., when $\theta = \varphi/2$.

$$v_{max} = \frac{4h\omega}{\varphi^2} \frac{\varphi}{2} = \frac{2h\omega}{\varphi} \tag{7.8}$$

During the second half of the follower motion, the follower is decelerated at constant rate so that the velocity reduces to zero at the end.

It can be observed from the plots shown in Fig. 7.18 that there are abrupt changes in the acceleration at the beginning, midway and the end of the follower motion. At midway, an infinite jerk is produced. Thus, this programme of the follower is adopted only up to moderate speeds.

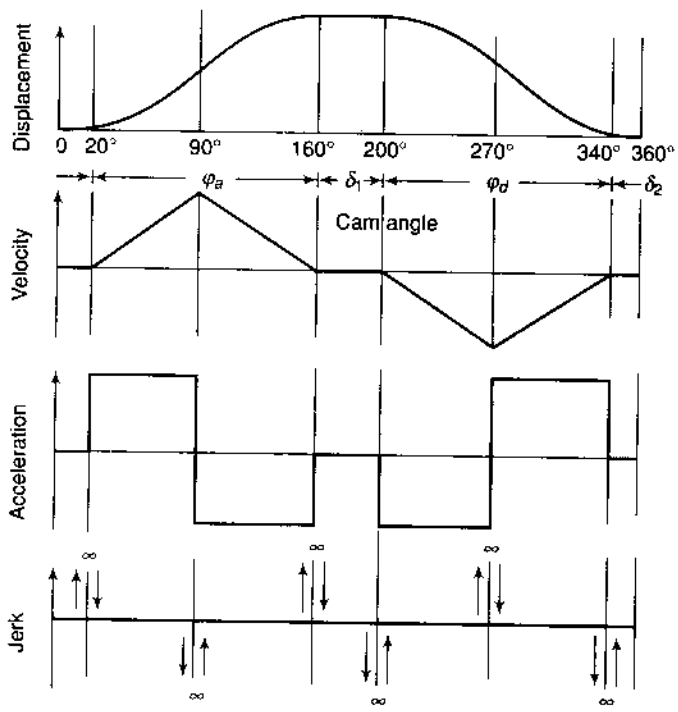

Fig. 7.18

3. Constant Velocity

Constant velocity of the follower implies that the displacement of the follower is proportional to the cam displacement and the slope of the displacement curve is constant (Fig. 7.19).

Displacement of the follower for the

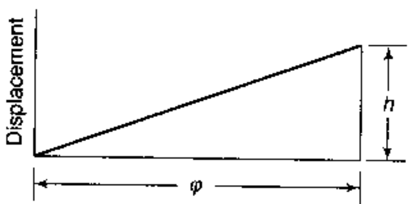

Fig. 7.19

angular displacement θ of the cam is given by

$$s = h\frac{\theta}{\varphi} = h\frac{\omega t}{\varphi} \tag{7.9}$$

$$v = \frac{ds}{dt} = \frac{h\omega}{\varphi}\text{ constant} \tag{7.10}$$

$$f = \frac{dv}{dt} = 0 \tag{7.11}$$

As seen in the plots of Fig. 7.20(a), though acceleration is zero during the rise or the fall of the follower, it is infinite at the beginning and end of the motion as there are abrupt changes in velocity at these points. This results in infinite inertia forces and thus is not suitable from the practical point of view.

A modified programme for the follower motion can be evolved in which the accelerations are reduced to finite values. This can be done by rounding the sharp corners of the displacement curve so that the velocity changes are gradual at the beginning and end of the follower motion. During these periods, the acceleration may be assumed to be constant and of finite values. A modified constant velocity programme is shown in Fig. 7.20(b).

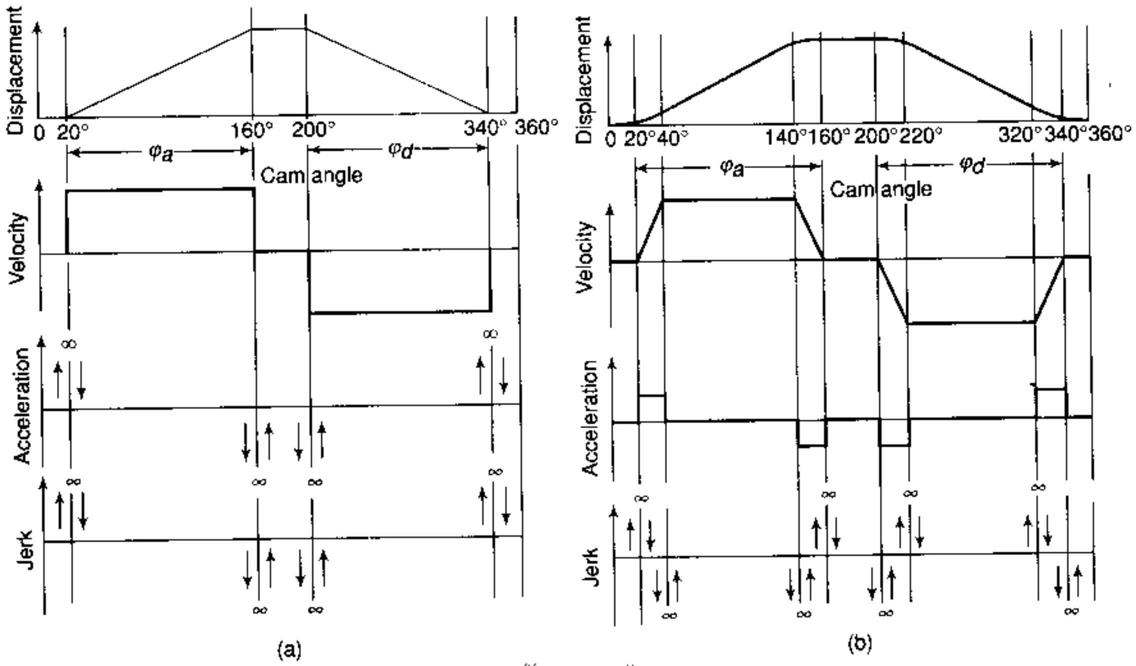

Fig. 7.20

4. Cycloidal

A cycloidal is the locus of a point on a circle rolling on a straight line.

Construction (Refer Fig. 7.21)

(i) Divide the cam displacement interval into n equal parts (n even).

(ii) Draw the diagonal of the diagram and extend it below.

(iii) Draw a circle with the centre anywhere on the lower portion of the diagonal such that its circumference is equal to the follower displacement, i.e., $2\pi r = h$ or $r = h/2\pi$.

(iv) Divide the circle into n equal arcs and number them as shown in the diagram.

(v) Project the circle points to its vertical diameter and then in a direction parallel to the diagonal of the diagram to the corresponding ordinates.

Joining the points with a curve gives the required cycloidal. Mathematically, a cycloidal is expressed by

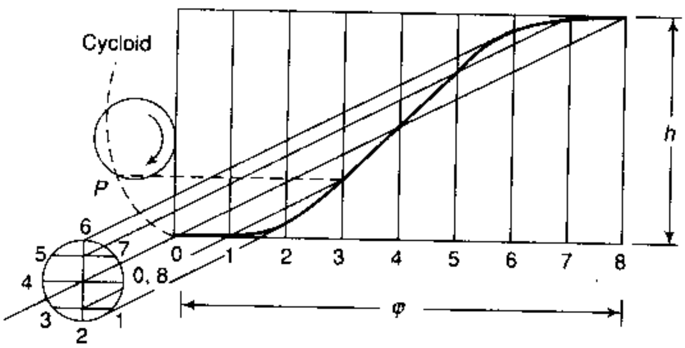

Fig. 7.21

$$s = \frac{h}{\pi}\left(\frac{\pi\theta}{\varphi} - \frac{1}{2}\sin\frac{2\pi\theta}{\varphi}\right) \tag{7.12}$$

$$v = \frac{ds}{dt} = \frac{ds}{d\theta}\frac{d\theta}{dt}$$

$$= \left[\frac{h}{\varphi} - \frac{h}{2\pi}\frac{2\pi}{\varphi}\cos\frac{2\pi\theta}{\varphi}\right]\omega$$

$$= \frac{h\omega}{\varphi} - \frac{h\omega}{\varphi}\cos\frac{2\pi\theta}{\varphi} \tag{7.13}$$

$$= \frac{h\omega}{\varphi}\left(1 - \cos\frac{2\pi\theta}{\varphi}\right)$$

$$v_{max} = \frac{2h\omega}{\varphi} \text{ at } \theta = \frac{\varphi}{2} \tag{7.14}$$

$$f = \frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt}$$

$$= \left[\frac{h\omega}{\varphi}\frac{2\pi}{\varphi}\sin\frac{2\pi\theta}{\varphi}\right]\omega$$

$$= \frac{2h\pi\omega^2}{\varphi^2}\sin\frac{2\pi\theta}{\varphi} \tag{7.15}$$

$$f_{max} = \frac{2h\pi\omega^2}{\varphi^2} \text{ at } \theta = \frac{\varphi}{4} \tag{7.16}$$

From the plots of Fig. 7.22, it is observed that there are no abrupt changes in the velocity and the acceleration at any stage of the motion. Thus, it is the most ideal programme for high-speed follower motion.

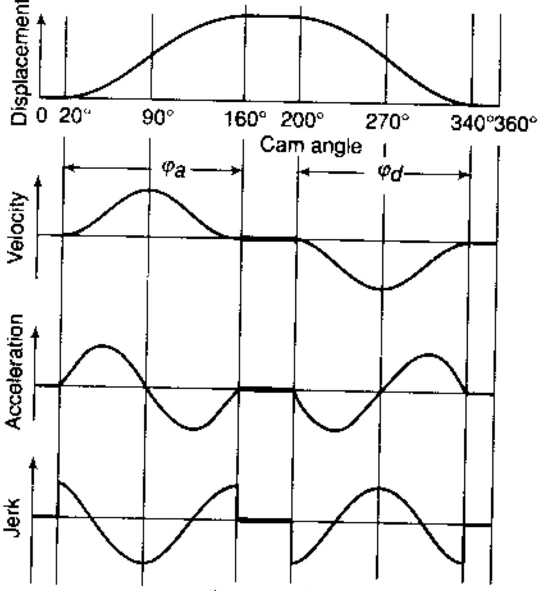

Fig. 7.22

7.9 LAYOUT OF CAM PROFILES

A cam profile is constructed on the principle of kinematic inversion, i.e., considering the cam to be stationary and the follower to be rotating about it in the opposite direction of the cam rotation. In general, the following procedure is adopted for laying out the profile for a reciprocating follower:

1. Draw the displacement diagram of the follower according to the given follower motion by dividing the cam displacement interval into n equal parts as has been discussed in the previous section. The usual number taken is 6, 8, 10 or 12 depending upon the angular displacement and convenience. Remember that the scale of the displacement interval does not affect the cam profile whereas the follower displacement does.
2. Draw the prime circle of the cam with radius
   (a) r_c if it is a knife-edge or mushroom follower (Ex. 7.1, 7.2 and 7.5)
   (b) $r_c + r_r$ if it is a roller follower (Ex. 7.3 and 7.4)
3. Divide the prime circle into segments as follows:
   (a) In case of a radial follower, divide the circle from the vertical position indicating the angles of ascent, dwell period and angle of descent, etc., in the opposite direction of the cam rotation (Ex. 7.1, 7.3 and 7.5).
   (b) In case of an offset follower, draw another circle with radius equal to the offset of the follower and assume the initial position on the prime circle where the tangent to the horizontal radius of the circle meets the prime circle (Ex. 7.2 and 7.4).
4. Further, divide each segment of ascent and descent into the same number of angular parts as is done in the displacement diagram.
5. On the radial lines produced, mark distances equal to the lift of the follower beyond the circumference of the prime circle (Ex. 7.1, 7.3 and 7.5). In case of offset follower, the distances are marked on the tangents drawn to the circle with radius equal to the offset (Ex. 7.2 and 7.4). It can be visualized that with rotation of the cam, each radial or tangential line so obtained merges with the axis of the follower at successive intervals of time and the marked points are the various positions of the tracing point of the follower.
6. Obtain the cam profile as follows:
   (a) For a knife-edge follower, draw a smooth curve passing through the marked points which is the required cam profile (Ex. 7.1 and 7.2).
   (b) In case of a roller follower, draw a series of arcs of radii equal to r_r on the inner side and draw a smooth curve tangential to all the arcs to get the required cam profile (Ex. 7.3 and 7.4).
   (c) For a mushroom follower, draw the follower in all the positions by drawing perpendiculars to the radial or tangent lines and draw a smooth curve tangential to the flat-faces of the follower representing the cam profile (Ex. 7.5).

Example 7.1 *Draw the profile of a cam operating a knife-edge follower having a lift of 30 mm. The cam raises the follower with SHM for 150° of the rotation followed by a period of dwell for 60°. The follower descends for the next 100° rotation of the cam with uniform velocity, again followed by a dwell period. The cam rotates at a uniform velocity of 120 rpm and has a least radius of 20 mm. What will be the maximum velocity and acceleration of the follower during the lift and the return?*

Solution:

$h = 30$ mm $\quad \varphi_a = 150°$

$N = 120$ rpm $\quad \delta_1 = 60°$

$r_c = 20$ mm $\quad \varphi_d = 100°$

$\delta_2 = (360° - 150° - 100° - 60°) = 50°$

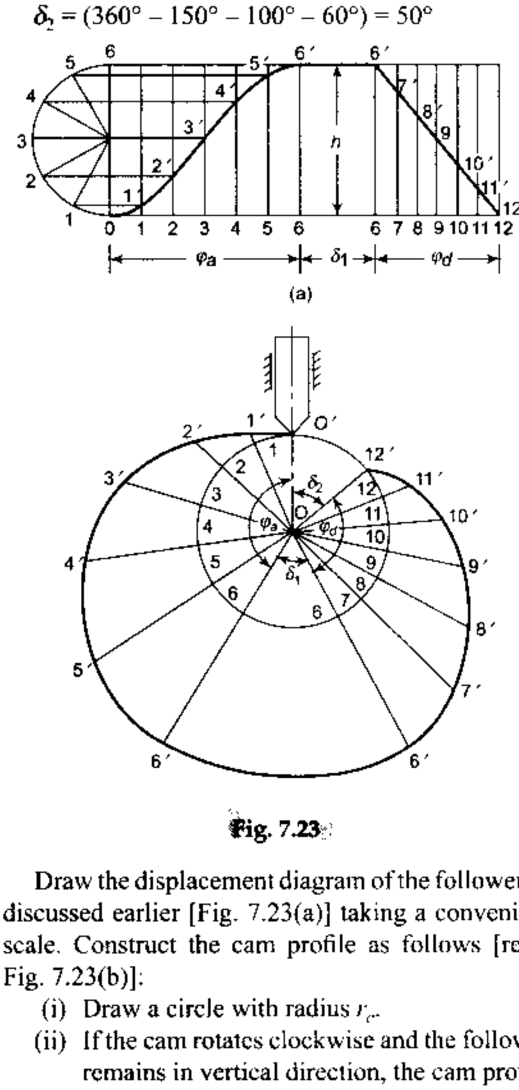

Fig. 7.23

Draw the displacement diagram of the follower as discussed earlier [Fig. 7.23(a)] taking a convenient scale. Construct the cam profile as follows [refer Fig. 7.23(b)]:

(i) Draw a circle with radius r_c.

(ii) If the cam rotates clockwise and the follower remains in vertical direction, the cam profile can be drawn by assuming that the cam is stationary and the follower rotates about the cam in the counter-clockwise direction.
From the vertical position, mark angles φ_a, δ_1, φ_d, and δ_2 in the counter-clockwise direction, representing angles of ascent, rest or dwell, descent and rest respectively.

(iii) Divide the angles φ_a and φ_d into same number of parts as is done in the displacement diagram. In this case, each has been divided into 6 equal parts.

(iv) Draw radial lines O-1, O-2, O-3, etc., O-1 represents that after an interval of $\varphi_d/6$ of the cam rotation in the clockwise direction it will take the vertical position of O-O'.

(v) On the radial lines produced, take distances equal to the lift of the follower beyond the circumference of the circle with radius r_c, i.e., 1-1′, 2-2′, 3-3′, etc.

(vi) Draw a smooth curve passing through O', 1′, 2′,...., 10′, 11′ and 12′. Draw an arc of radius O-6′ for the dwell period δ_1.

During ascent

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 120}{60} = 12.57 \text{ rad/s}$$

$$v_{max} = \frac{h}{2}\frac{\pi\omega}{\varphi_a} \qquad \text{[refer Eq. (7.3)]}$$

or

$$v_{max} = \frac{30}{2} \times \frac{\pi \times 12.57}{150 \times \frac{\pi}{180}} = 226.3 \text{ mm/s}$$

$$f_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\varphi_a}\right)^2 \qquad \text{[refer Eq. (7.5)]}$$

or

$$f_{max} = \frac{30}{2} \times \left(\frac{\pi \times 12.57}{150 \times \frac{\pi}{180}}\right)^2$$

$= 3413$ mm/s^2 or $\underline{3.413 \text{ m/s}^2}$

During desent

$$v_{max} = h\frac{\omega}{\varphi_d} \qquad \text{[refer Eq. (7.10)]}$$

$$v_{max} = 30 \times \frac{12.57}{100 \times \frac{\pi}{180}} = \underline{216 \text{ mm/s}}$$

$$f_{max} = f = 0$$

Note that to draw the cam profile, it is not necessary that the interval δ_1 is taken in the displacement diagram. Also, the scales of φ_a and φ_d can be taken different and of any magnitudes.

Example 7.2 *A cam with a minimum radius of 25 mm is to be designed for a knife-edge follower with the following data:*

- *To raise the follower through 35 mm during 60° rotation of the cam*
- *Dwell for next 40° of the cam rotation*
- *Descending of the follower during the next 90° of the cam rotation*
- *Dwell during the rest of the cam rotation*

Draw the profile of the cam if the ascending and descending of the cam is with simple harmonic motion and the line of stroke of the follower is offset 10 mm from the axis of the cam shaft.

What is the maximum velocity and acceleration of the follower during the ascent and the descent if the cam rotates at 150 rpm?

Solution

$h = 35$ mm $\qquad \varphi_a = 60°$

$N = 150$ rpm $\qquad \delta_1 = 40°$

$r_c = 25$ mm $\qquad \varphi_d = 90°$

$x = 10$ mm

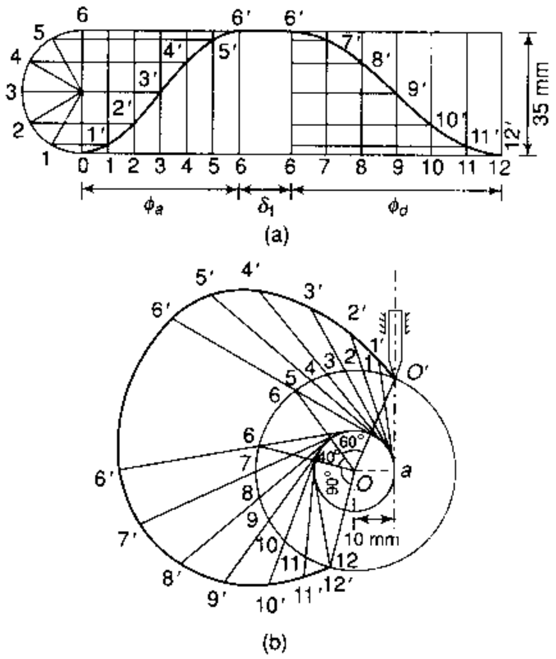

Fig. 7.24

Draw the displacement diagram of the follower as discussed earlier [Fig. 7.24(a)]. Construct the cam profile as follows [refer Fig. 7.24(b)]:

(i) Draw a circle with radius r_c (= 25 mm).

(ii) Draw another circle concentric with the previous circle with radius x (= 10 mm). If the cam is assumed stationary, the follower will be tangential to this circle in all the positions. Let the initial position be a-O'.

(iii) Join O-O'. Divide the circle of radius r_c into four parts as usual with angles φ_a, δ_1, φ_d and δ_2 starting from O-O'.

(iv) Divide the angles φ_a and φ_d into same number of parts as is done in the displacement diagram and obtain the points 1, 2, 3, etc., on the circumference of circle with radius r_c.

(v) Draw tangents to the circle with radius x from the points 1, 2, 3, etc.

(vi) On the extension of the tangent lines, mark the distances from the displacement diagram.

(vii) Draw a smooth curve through O', 1′, 2′, etc. This is the required pitch curve.

During ascent $\quad \omega = \dfrac{2\pi \times 150}{60} = 5\pi$ rad/s

$$v_{max} = \frac{h}{2}\cdot\frac{\pi\omega}{\varphi_a} \qquad \text{[refer Eq. (7.3)]}$$

or $$v_{max} = \frac{35}{2} \times \frac{\pi \times 5\pi}{60 \times \frac{\pi}{180}} = \underline{824.7 \text{ mm/s}}$$

$$f_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\varphi_a}\right)^2 \qquad \text{[refer Eq. (7.5)]}$$

or $$f_{max} = \frac{35}{2} \times \left(\frac{\pi \times 5\pi}{60 \times \frac{\pi}{180}}\right)^2$$

$$= 38\,862 \text{ mm/s}^2 \text{ or } \underline{38.882 \text{ m/s}^2}$$

During descent

$$v_{max} = \frac{35}{2} \times \frac{\pi \times 5\pi}{90 \times \frac{\pi}{180}} = \underline{549.8 \text{ mm/s}}$$

$$f_{max} = \frac{35}{2} \times \left(\frac{\pi \times 5\pi}{90 \times \frac{\pi}{180}}\right)^2$$

$$= 17\,272 \text{ mm/s}^2 \text{ or } 17.272 \text{ m/s}^2$$

Example 7.3 *A cam is to give the following motion to a knife-edged follower:*

- *To raise the follower through 30 mm with uniform acceleration and deceleration during 120° rotation of the cam*
- *Dwell for next 30° of the cam rotation*
- *To lower the follower with simple harmonic motion during the next 90° rotation of the cam*
- *Dwell for the rest of the cam rotation*

The cam has a minimum radius of 30 mm and rotates counter-clockwise at a uniform speed of 800 rpm. Draw the profile of the cam if the line of stroke of the follower passes through the axis of the cam shaft. Also, draw the displacement, velocity and the acceleration diagrams for the motion of the follower for one complete revolution of the cam indicating main values.

Solution

$h = 30$ mm, $r_c = 30$ mm, $\varphi_a = 120°$,

$\delta_1 = 30°$, $\varphi_d = 90°$, $N = 800$ rpm

$\delta_2 = 360° - 120° - 30° - 90° = 120°$

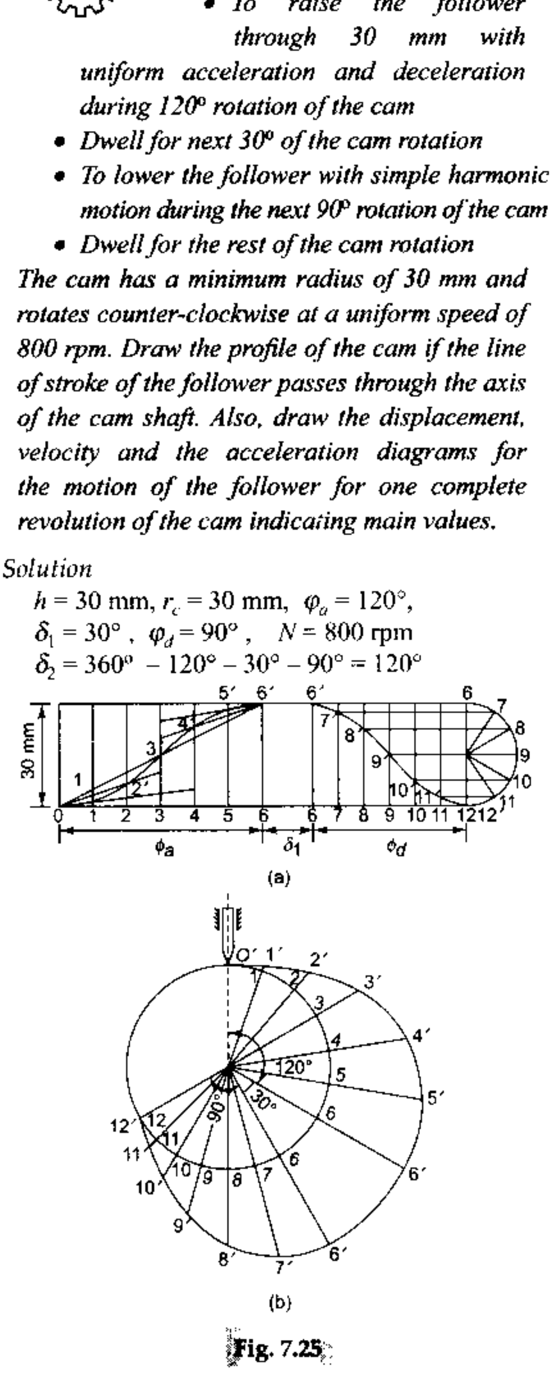

Fig. 7.25

Draw the displacement diagram of the follower as shown in Fig. 7.25(a). As the rotation of the cam shaft is counter-clockwise, the cam profile is to be drawn assuming the cam to be stationary and the follower rotating clockwise about the cam. Construct the cam profile as described below [Fig. 7.25(b)]:

(i) Draw a circle with radius r_c.

(ii) From the vertical position, mark angles φ_a, δ_1, φ_d and δ_2 in the clockwise direction.

(iii) Divide the angles φ_a and φ_d into same number of parts as is done in the displacement diagram. In this case, φ_a as well as φ_d have been divided into 6 equal parts.

(iv) On the radial lines produced, mark the distances from the displacement diagram.

(v) Draw a smooth curve tangential to end points of all the radial lines to obtain the required cam profile.

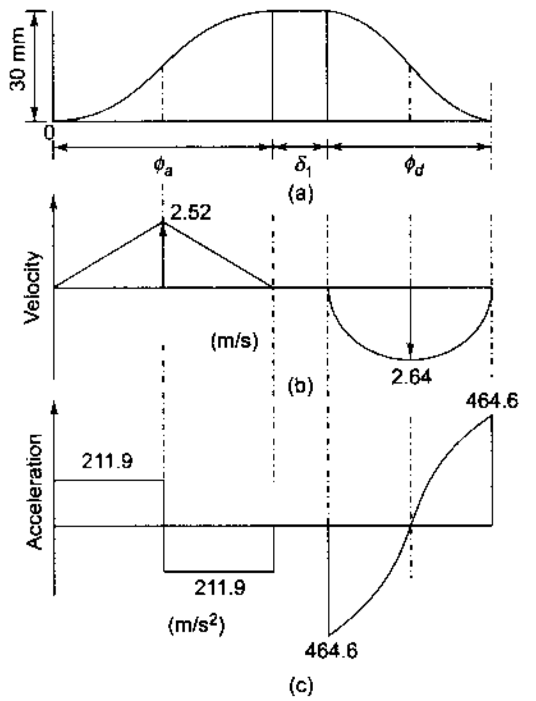

Fig. 7.26

The displacement diagram is reproduced in Fig. 7.26(a). The velocity and acceleration diagrams are to be drawn below this figure.

$$\omega = \frac{2\pi \times 840}{60} = 88 \text{ rad/s}$$

During ascent

During the ascent period, the acceleration and the deceleration are uniform. Thus, the velocity is linear and is given by

$$v = \frac{4h\omega}{\varphi_a^2}.\theta \qquad \text{[Eq. (7.7a)]}$$

The maximum velocity is at the end of the acceleration period, i.e., when $\theta = \varphi_{a/2}$.

$$\therefore \quad v_{max} = 2h\frac{\omega}{\varphi_a}$$

$$= 2 \times 0.03 \times \frac{88}{120\pi/180} = \underline{2.52 \text{ m/s}}$$

The plot of velocity variation during the ascent period is shown in Fig. 7.26(b).

$$f_{uniform} = \frac{4h\omega^2}{\varphi_a^2} \qquad \text{[Eq. (7.6a)]}$$

$$\text{or} \quad f_{uniform} = \frac{4 \times 0.03 \times 88^2}{(120\pi/180)^2} = 211.9 \text{ m/s}^2$$

This has been shown in Fig. 7.26(c).

During descent

During descent, it is simple harmonic motion. The variation of velocity is give by

$$v = \frac{h}{2}\frac{\pi\omega}{\varphi_d}\sin\frac{\pi\theta}{\varphi_d} \qquad \text{[Eq. (7.2)]}$$

Maximum value is at $\theta = \varphi_d / 2$,

$$v_{max} = \frac{h}{2}\frac{\pi\omega}{\varphi_d}$$

$$\therefore \quad = \frac{0.03}{2} \times \frac{\pi \times 88}{90\pi/180} = 2.64 \text{ mm/s}$$

The plot of velocity variation during the descent period is shown in Fig. 7.26(b).

The acceleration variation is given by,

$$f = \frac{h}{2}\left(\frac{\pi\omega}{\varphi}\right)^2 \cos\frac{\pi\theta}{\varphi} \qquad \text{[Eq. (7.4)]}$$

It is maximum at $\theta = 0$, i.e.,

$$f_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\varphi_d}\right)^2$$

$$= \frac{0.03}{2} \times \left(\frac{\pi \times 88}{90\pi/180}\right)^2 = \underline{464.6 \text{ m/s}^2}$$

This variation is shown in Fig. 7.26(c).

Example 7.4 *Draw the profile of a cam operating a roller reciprocating follower and with the following data:*

Minimum radius of cam = 25 mm
Lift = 30 mm
Roller diameter = 15 mm

The cam lifts the follower for 120° with SHM followed by a dwell period of 30°. Then the follower lowers down during 150° of the cam rotation with uniform acceleration and deceleration followed by a dwell period. If the cam rotates at a uniform speed of 150 rpm, calculate the maximum velocity and acceleration of the follower during the descent period.

Solution:

$h = 30$ mm	$\varphi_a = 120°$
$N = 150$ mm	$\delta_1 = 30°$
$r_c = 25$ mm	$\varphi_d = 150°$
$r_r = 7.5$ mm	$\delta_2 = 60°$

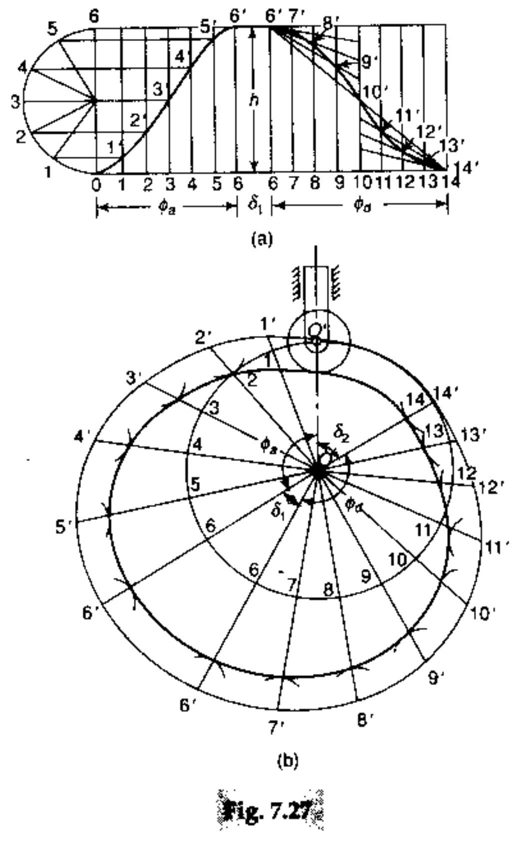

Fig. 7.27

Draw the displacement diagram of the follower as shown in Fig. 7.27(a). Construct the cam profile as described below [Fig. 7.27(b)].

(i) Draw a circle with radius $(r_c + r_r)$.

(ii) From the vertical position, mark angles φ_a, δ_1, φ_d and δ_2 in the counter-clockwise direction (assuming that the cam is to rotate in the clockwise direction).

(iii) Divide the angles φ_a and φ_d into the same number of parts as is done in the displacement diagram. In this case, φ_a has been divided into 6 equal parts whereas φ_d is divided into 8 equal parts.

(iv) On the radial lines produced, mark the distances from the displacement diagram.

(v) Draw a series of arcs of radii equal to r_r, as shown in the diagram from the points 1′, 2′, 3′, etc.

(vi) Draw a smooth curve tangential to all the arcs which is the required cam profile.

During the descent period, the acceleration and the deceleration are uniform. Therefore, the maximum velocity is at the end of the acceleration period.

$$v_{max} = 2h\frac{\omega}{\varphi_d} \qquad \text{Eq. (7.8)}$$

or

$$v_{max} = 2 \times 30 \times \frac{\dfrac{2\pi \times 150}{60}}{150 \times \dfrac{\pi}{180}} = \underline{360 \text{ m/s}}$$

$$f_{max} = f_{uniform} = \frac{4h\omega^2}{\varphi_d^2} \qquad \text{Eq. (7.6)}$$

$$f_{max} = \frac{4 \times 30 \times \left(\dfrac{2\pi \times 150}{60}\right)^2}{\left(150 \times \dfrac{\pi}{180}\right)^2} = 4320 \text{ mm/s}^2$$

or $\underline{4.32 \text{ m/s}^2}$

Example 7.5 *The following data relate to a cam profile in which the follower moves with uniform acceleration and deceleration during ascent and descent.*

Minimum radius of cam = 25 mm
Roller diameter = 7.5 mm
Lift = 28 mm
Offset of follower axis = 12 mm towards right
Angle of ascent = 60°
Angle of descent = 90°
Angle of dwell between ascent and descent = 45°
Speed of the cam = 200 rpm

Draw the profile of the cam and determine the maximum velocity and the uniform acceleration of the follower during the outstroke and the return stroke.

Solution:

$h = 28$ mm $\quad \varphi_a = 60°$
$r_c = 25$ mm $\quad \delta_1 = 45°$
$r_r = 7.5$ mm $\quad \varphi_d = 90°$
offset, $x = 12$ mm $\quad \delta_2 = (360° - 60° - 45° - 90°) = 165°$
$N = 200$ mm

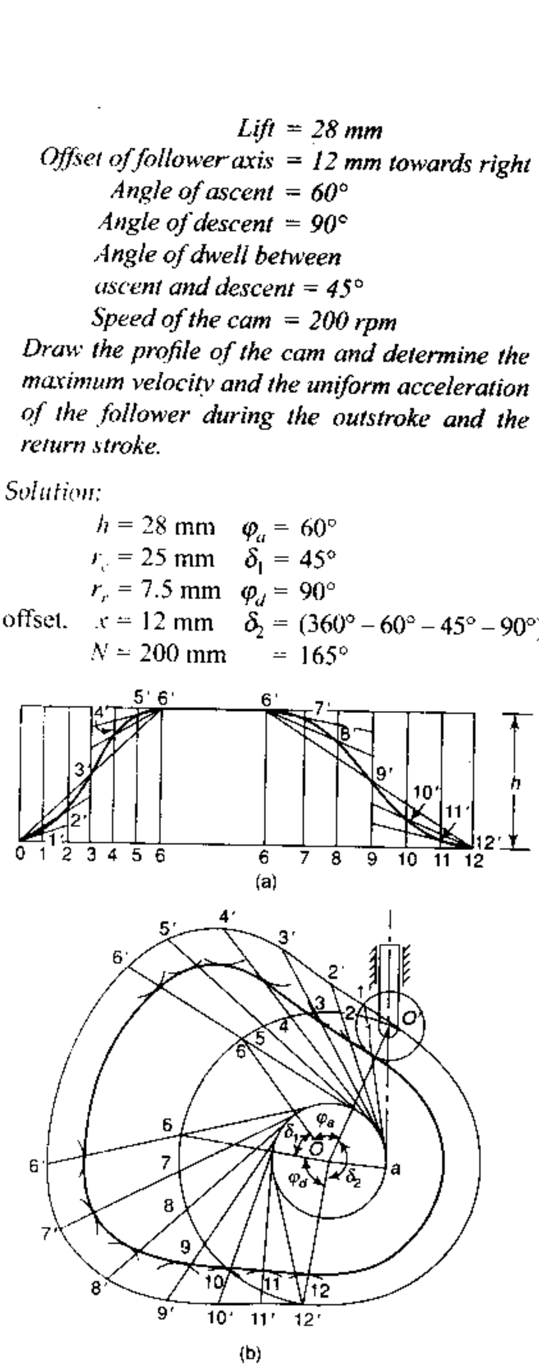

Fig. 7.28

From the given data, construct the displacement diagram as usual [Fig. 7.28(a)]. For the cam profile [Fig. 7.28(b)], the procedure is as follows:

(i) Draw a circle with radius $(r_c + r_r)$.

(ii) Draw another circle concentric with the previous circle with radius x. If the cam is assumed stationary, the follower will be tangential to this circle in all the positions. Let the initial position be a-O'.

(iii) Join O-O'. Divide the circle of radius $(r_c + r_r)$ into four parts as usual with angles φ_a, δ_1, φ_d and δ_2 starting from O-O'.

(iv) Divide the angles φ_a and φ_d into same number of parts as is done in the displacement diagram and obtain the points 1, 2, 3, etc., on the circumference of circle with radius $(r_c + r_r)$.

(v) Draw tangents to the circle with radius x from the points 1, 2, 3, etc.

(vi) On the extension of the tangent lines, mark the distances from the displacement diagram.

(vii) Draw a smooth curve through O', 1′, 2′, etc. This is the pitch curve.

(viii) With 1′, 2′, 3′, etc., as centres, draw a series of arcs of radii equal to r_r.

(ix) Draw a smooth curve tangential to all the arcs and obtain the required cam profile.

During outstroke

$$\omega = \frac{2\pi \times 200}{60} = 20.94 \text{ rad/s}$$

$$v_{max} = 2h\frac{\omega}{\varphi_a}$$
$$= 2 \times 28 \times \frac{20.94}{60 \times \pi / 180}$$
$$= 1120 \text{ mm/s or } 1.12 \text{ m/s}$$

$$f_{uniform} = \frac{4h\omega^2}{\varphi_a^2}$$
$$= \frac{4 \times 28 \times (20.94)^2}{\left(60 \times \frac{\pi}{180}\right)^2}$$
$$= 44\,800 \text{ mm/s}^2 \text{ or } 44.8 \text{ m/s}^2$$

During return stroke

$$v_{max} = \frac{2h\omega}{\varphi_d}$$
$$= \frac{2 \times 28 \times 20.94}{90 \times \pi / 180}$$
$$= 747 \text{ mm/s or } 0.747 \text{ m/s}$$

$$f_{uniform} = \frac{4 \times 28 \times (20.94)^2}{\left(90 \times \frac{\pi}{180}\right)^2}$$
$$= 19\,900 \text{ mm/s}^2 \text{ or } 19.9 \text{ m/s}^2$$

Example 7.6 *A flat-faced mushroom follower is operated by a uniformly rotating cam. The follower is raised through a distance of 25 mm in 120° rotation of the cam, remains at rest for the next 30° and is lowered during further 120° rotation of the cam. The raising of the follower takes place with cycloidal motion and the lowering with uniform acceleration and deceleration. However, the uniform acceleration is 2/3 of the uniform deceleration. The least radius of the cam is 25 mm which rotates at 300 rpm.*

Draw the cam profile and determine the values of the maximum velocity and maximum acceleration during rising, and maximum velocity and uniform acceleration and deceleration during lowering of the follower.

Solution:

$h = 25$ mm	$\varphi_a = 120°$
$r_c = 25$ mm	$\delta_1 = 30°$
$N = 300$ mm	$\varphi_d = 120°$
	$\delta_2 = 90°$

During the return stroke, as the uniform acceleration is 2/3 of the uniform deceleration, the uniform deceleration is 3/2 of the uniform acceleration.

Let the uniform acceleration be f so that the uniform deceleration be $(3/2)f$.

Time of acceleration

Final velocity,

$v = u + ft = ft$ (initial velocity is zero)

or $\quad t = \dfrac{v}{f}$

Time of deceleration

Initial velocity is v and final velocity zero.

Therefore,

$0 = v - (3/2)ft'$

where t' is the time of deceleration and negative sign due to deceleration.

or $$t' = \frac{2}{3} \cdot \frac{v}{f}$$

Thus, the time of deceleration is 2/3 of the time of acceleration.

Displacement

During acceleration, $s = ut + \frac{1}{2} ft^2$

$$= \frac{1}{2} ft^2 \quad \text{(i)}$$

During deceleration

$$= ft\left(\frac{2}{3}t\right) - \frac{1}{2}\left(\frac{3}{2}f\right)\left(\frac{2}{3}t\right)^2$$

[as initial velocity ft and time taken $(2/3)t$]

$$= \frac{2}{3} ft^2 - \frac{1}{3} ft^2$$

$$= \frac{2}{3}\left(\frac{1}{2} ft^2\right) \quad \text{(ii)}$$

Comparison of (i) and (ii), shows that the distance travelled during deceleration period is 2/3 of the distance travelled during acceleration.

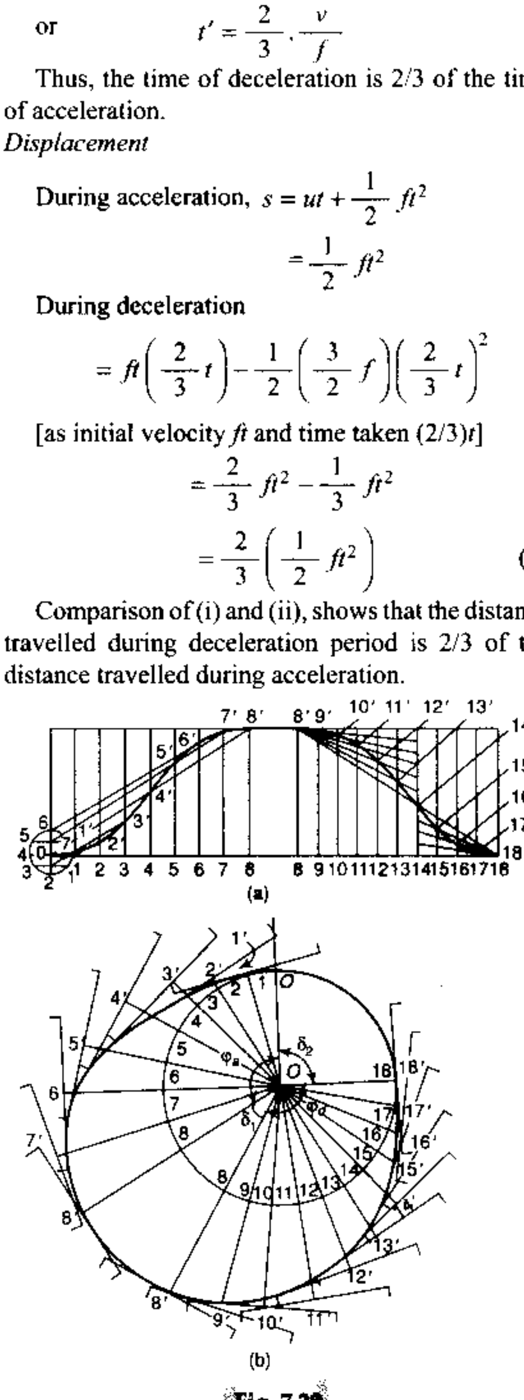

Fig. 7.29

The displacement diagram has been shown in Fig. 7.29(a). Note that during the return stroke, the time of acceleration and the displacement are 3/2 times of the corresponding values during the deceleration. (Time is measured along the x-axis and displacement along the y-axis). Thus, the time of acceleration is 3/5 of the total time of return and the displacement is 3/5 of the total displacement.

To draw the cam profile, proceed as follows [Fig. 7.29(b)]:

(i) Draw a circle with radius r_c.

(ii) Take angles φ_a, δ_1, φ_d and δ_2 as before (in the counter-clockwise direction if the cam rotation is assumed clockwise).

(iii) Divide φ_a and φ_d into same number of parts as in the displacement diagram.

(iv) Draw radial lines and on them mark the distances 1-1′, 2-2′, 3-3′, etc.

(v) Draw the follower in all the positions by drawing perpendiculars to the radial lines at 1′, 2′, 3′, etc. In all the positions, the axis of the follower passes through the centre O.

(vi) Draw a curve tangential to the flat-faces of the follower representing the cam profile.

Remember that in case of mushroom followers, the contact point will rarely lie on the axis of the follower.

During ascent

$$v_{max} = \frac{2h\omega}{\varphi_a} \quad \text{Eq. (7.14)}$$

where $\omega = \frac{2\pi \times 300}{60} = 31.4$ rad/s

$$v_{max} = \frac{2 \times 25 \times 31.4}{120 \times \pi / 180} = 750 \text{ mm/s or } 0.75 \text{ m/s}$$

$$f_{max} = \frac{2h\pi\omega^2}{\varphi_a^2} \quad \text{Eq. (7.16)}$$

$$f_{max} = \frac{2 \times 25 \times \pi \times (31.4)^2}{(120 \times \pi / 180)^2}$$

$= 35\,310$ mm/s^2 or 35.31 m/s^2

During descent

$$v = ft \quad \text{Eq. (7.7)}$$

or $$v = \left(\frac{2s}{t^2}\right)t = \frac{2s}{t} \quad \text{[Refer Eq. (7.6)]}$$

v will be maximum at the end of the acceleration period. At the end of the acceleration period,

$$s = \frac{3}{5} \times 25 = 15 \text{ mm}$$

and the time taken to travel this distance is found as under,

Time for 300 rev. = 60 s

$$\text{Time for 1 rev. } (= 360°) = \frac{60}{300} = 0.2 \text{ s}$$

$$\text{Time for } \left(\frac{3}{5} \times 120°\right) = \frac{0.2}{360} \times 72 = 0.04 \text{ s}$$

$$v_{max} = \frac{2 \times 15}{0.04} = \underline{750 \text{ mm/s}^2} \text{ or } \underline{0.075 \text{ m/s}^2}$$

Uniform acceleration

$$= \frac{v_{max}}{\text{time}} = \frac{0.75}{0.04} = \underline{18.75 \text{ m/s}^2}$$

Uniform deceleration

$$= 18.75 \times \frac{3}{2} = \underline{28.13 \text{ m/s}^2}$$

Example 7.7 ***The following data relate to a cam operating an oscillating roller follower:***

Minimum radius of cam	=	*44 mm*
Diameter of roller	=	*14 mm*
Length of the follower arm	=	*40 mm*
Distance of fulcrum centre from cam centre	=	*50 mm*
Angle of ascent	=	*75°*
Angle of descent	=	*105°*
Angle of dwell for follower in the highest position	=	*60°*
Angle of oscillation of follower	=	*28°*

Draw the profile of the cam if the ascent and descent both take place with SHM.

Solution:

$r_c = 22$ mm $\theta = 28°$

$r_r = 7$ mm $\varphi_a = 75°$

Follower arm length = 40 mm $\delta_1 = 60°$

$h = \theta \times$ arm length $\varphi_d = 105°$

$$= \left(28° \times \frac{\pi}{180°}\right) \times 40 \qquad \delta_2 = 120°$$

= 19.5 mm

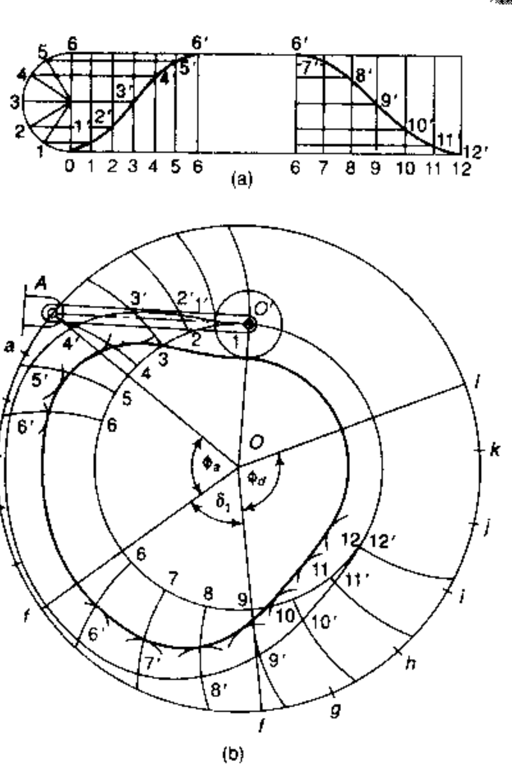

Fig. 7.30

The displacement diagram has been shown in Fig. 7.30(a). To draw the cam profile, proceed as follows:

(i) Draw a circle with radius $(r_c + r_r)$ [Fig. 7.30(b)].

(ii) Assuming the initial position of the roller centre vertically above the cam centre O, locate the fulcrum centre as its distances from the cam centre and the roller centre (equal to length of follower arm) are known.

(iii) Draw a circle with radius OA and centre at O.

(iv) On the circle through A, starting form OA, take angles φ_a, δ_1 and φ_d as usual.

(v) Divide the angles φ_a and φ_d into same number of parts as is done in the displacement diagram and obtain the points a, b, c, d, etc., on this circle through A.

(vi) With centres A, a, b, etc., draw arcs with radii equal to length of the arm.

(vii) Mark distances 1-1′, 2-2′, 3-3′, etc., on these arcs as shown in the diagram. It is on the assumption that for small angular displacements, the linear displacements on the arcs and on the straight lines are the same.

(viii) With 1′, 2′, 3′, etc., draw a series of arcs of radii equal of r_r.

(ix) Draw a smooth curve tangential to all the arcs and obtain the required cam profile.

7.10 CAMS WITH SPECIFIED CONTOURS

It is always desired that a cam is made to provide a smooth motion of the follower and for that a follower motion programme is always selected first. However, sometimes it becomes difficult to manufacture the cams in large quantities of the specified contours. Under such circumstances, it becomes necessary that the cam is designed first and then some improvements are made in that if possible. Such cams are generally made up of some combination of curves such as straight lines, circular arcs, etc. In the present section, some cams with specified contours are analysed.

1. Tangent Cam (with Roller Follower)

A tangent cam is symmetrical about the centre line. It has straight flanks (such as AK in Fig. 7.31) with a circular nose. The centre of the cam is at O and that of the nose at Q. A tangent cam is used with a roller cam since there is no meaning of using flat-faced followers with straight flanks.

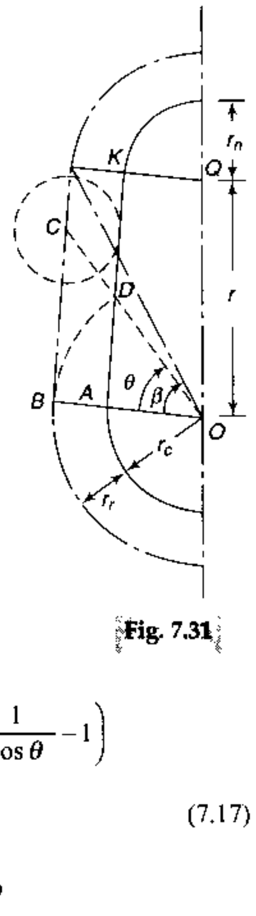

Fig. 7.31

Let r_c = least radius of cam
r_n = radius of nose
r_r = radius of roller
r = distance between the cam and the nose centres.

Roller on the Flank When the roller is on the straight flank, the centre of the roller is at C on the pitch profile as shown in Fig. 7.31.

Let θ = angle turned by the cam from the beginning of the follower motion

Let,
$$x = OC - OD = OC - OB = \frac{OB}{\cos\theta} - OB = OB\left(\frac{1}{\cos\theta} - 1\right)$$

or
$$x = (r_c + r_r)\left(\frac{1}{\cos\theta} - 1\right) \tag{7.17}$$

$$v = \frac{dx}{dt} = \frac{dx}{d\theta}\cdot\frac{d\theta}{dt} = (r_c + r_r)\left(\frac{\sin\theta}{\cos^2\theta} - 0\right)\omega$$

$$= \omega\,(r_c + r_r)\frac{\sin\theta}{\cos^2\theta} \tag{7.18}$$

$\sin\theta$ increases with the increase in θ whereas $\cos\theta$ decreases. Hence the velocity increases with θ and it is maximum when θ is maximum. This will happen when the point of contact leaves the straight flank.

Let β = angle turned by the cam when the roller leaves the flank.

$$\therefore \quad v_{max} = \omega\,(r_c + r_r)\frac{\sin\beta}{\cos^2\beta} \text{ and } v_{min} = 0 \text{ at } \theta = 0$$

$$f = \frac{dv}{dt} = \frac{dv}{d\theta} \times \frac{d\theta}{dt}$$

$$= \omega (r_c + r_r) \left[\frac{\cos^2\theta \cos\theta - \sin\theta(-2\sin\theta\cos\theta)}{\cos^4\theta} \right] \omega$$

$$= \omega^2 (r_c + r_r) \left[\frac{\cos^2\theta + 2\sin^2\theta}{\cos^3\theta} \right]$$

$$= \omega^2 (r_c + r_r) \left[\frac{2(\cos^2\theta + \sin^2\theta) - \cos^2\theta}{\cos^3\theta} \right] \quad (7.19)$$

$$= \frac{\omega^2 (r_c + r_r)(2 - \cos^2\theta)}{\cos^3\theta}$$

Acceleration is minimum when $\frac{2-\cos^2\theta}{\cos^3\theta}$ is minimum, i.e., when $(2-\cos^2\theta)$ is minimum and $\cos^3\theta$ is maximum or when $\cos\theta$ is maximum.

This is possible when $\theta = 0°$ or when the roller touches the straight flank.

$$f_{min} = \omega^2 (r_c + r_r) \quad (7.20)$$

Roller on the Nose Let the centre of the roller be at the point C on the pitch profile over the nose. Let QN be the perpendicular to OC (Fig. 7.32).

$$x = OC - OD$$

$$= (ON + NC) - OB$$

$$= OQ\cos\varphi + CQ\cos\psi - OB$$

$$= OQ\cos\varphi + CQ\sqrt{1-\sin^2\psi} - OB$$

$$= OQ\cos\varphi + CQ\sqrt{1 - \frac{(NQ)^2}{(CQ)^2}} - OB$$

$$= r\cos(\alpha-\theta) + \sqrt{l^2 - r^2\sin^2(\alpha-\theta)} - n \quad (7.21)$$

where $l = CQ = r_n + r_r$

and $n = OB = r_c + r_r$

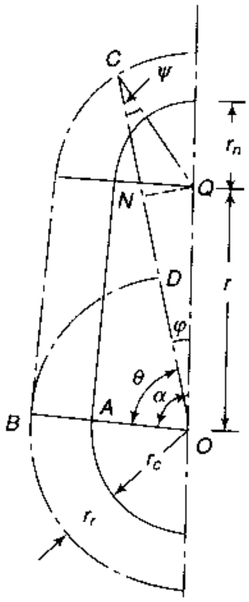

Fig. 7.32

$$v = \frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt}$$

$$= \left[\frac{d}{d\theta}\{r\cos(\alpha-\theta) + \sqrt{(l^2 - r^2\sin^2(\alpha-\theta))} - n\} \right]\omega$$

$$= \omega\left[-r\sin(\alpha-\theta)(-1) + \frac{1}{2}\frac{r^2 2\sin(\alpha-\theta)\cos(\alpha-\theta)}{\sqrt{(l^2 - r^2\sin^2(\alpha-\theta))}} \right]$$

$$= \omega r\left[\sin(\alpha-\theta) + \frac{r\sin 2(\alpha-\theta)}{2\sqrt{(l^2 - r^2\sin^2(\alpha-\theta))}} \right] \quad (7.22)$$

$$f = \frac{dv}{dt} = \frac{dv}{d\theta}\,\frac{d\theta}{dt} = \omega r \frac{d}{d\theta}\left[\sin(\alpha-\theta) + \frac{r}{2}\,\frac{\sin 2(\alpha-\theta)}{\sqrt{(l^2 - r^2\sin^2(\alpha-\theta))}}\right]\omega$$

$$= \omega^2 r \frac{d}{d\theta}\left[\sin(\alpha-\theta) + \frac{\omega^2 r^2}{2}\,\frac{d}{d\theta}[\sin 2(\alpha-\theta)]\{l^2 - r^2\sin^2(\alpha-\theta)\}^{-1/2}\right]$$

$$= \omega^2 r[\cos(\alpha-\theta)(-1)] + \frac{\omega^2 r^2}{2}[\sin 2(\alpha-\theta)\left(-\frac{1}{2}\right)$$

$$\{l^2 - r^2\sin^2(\alpha-\theta)\}^{-3/2}(-r^2)\{2\sin(\alpha-\theta)\cos(\alpha-\theta)(-1)\}$$
$$+\{l^2 - r^2\sin^2(\alpha-\theta)\}^{-1/2}\,2\cos 2(\alpha-\theta)(-1)]$$

$$= \omega^2 r\left[-\cos(\alpha-\theta) - \frac{r^3\sin^2 2(\alpha-\theta)}{4[l^2 - r^2\sin^2(\alpha-\theta)]^{3/2}} - \frac{r\cos 2(\alpha-\theta)}{\sqrt{l^2 - r^2\sin^2(\alpha-\theta)}}\right] \tag{7.23}$$

2. Circular Arc (Convex) Cam (with Flat-faced Follower)

A circular arc cam is made up of three arcs of different radii (Fig. 7.33). In such cams, the acceleration may change abruptly at the blending points due to instantaneous change in the radius of curvature.

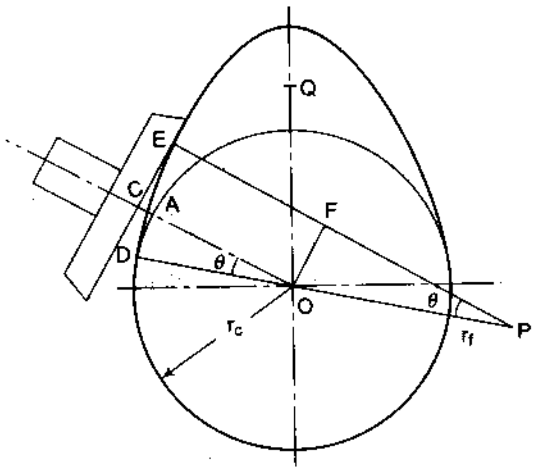

Fig. 7.33

Follower Touching Circular Flank When the flat-faced follower touches point E on the circular flank, it is lifted through a distance AC.

Let P be the centre of the circular arc of the flank and r_f the radius of the circular flank ($= PD = PE$).

$$x = OC - OA = EF - OA = (PE - PF) - OA$$

$$= PE - OP\cos\theta - OA$$

$$= PE - (PD - OD)\cos\theta - OA$$

or

$$x = r_f - (r_f - r_c)\cos\theta - r_c$$

$$= r_f - r_f\cos\theta + r_c\cos\theta - r_c$$

$$= r_f(1-\cos\theta) - r_c(1-\cos\theta)$$

$$= (r_f - r_c)(1-\cos\theta) \tag{7.24}$$

$$v = \frac{dx}{dt} = \frac{dx}{d\theta}\,\frac{d\theta}{dt}$$

$$= [(r_f - r_c)(\sin\theta)]\omega$$

$$= \omega(r_f - r_c)\sin\theta \tag{7.25}$$

It is zero when $\theta = 0$, i.e., when the follower starts ascending. It increases with θ and is maximum when θ is maximum or when the follower leaves the flank and $\theta = \beta$.

$$v_{max} = \omega(r_f - r_c)\sin\beta$$

$$f = \frac{dv}{dt} = \frac{dv}{d\theta}\,\frac{d\theta}{dt} = \left[(r_f - r_c)\cos\theta\right]\omega = \omega^2(r_f - r_c)\cos\theta \tag{7.26}$$

It is maximum at the beginning when $\theta = 0$, i.e when the rise commences.

$$f_{\max} = \omega^2 (r_f - r_c)$$
$$f_{\min} = \omega^2 (r_f - r_c) \cos \beta$$

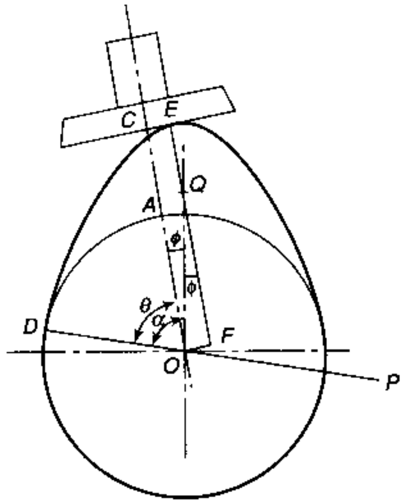

Fig. 7.34

Follower on the Nose (Fig. 7.34)

$$x = OC - OA = EF - OA$$
$$= QE + QF - OA$$
$$= QE + OQ \cos\varphi - OA$$
$$= r_n + r \cos(\alpha - \theta) - r_c$$
$$= r_n - r_c + r \cos(\alpha - \theta) \qquad (7.27)$$
$$v = \frac{dx}{dt} = \frac{dx}{d\theta} \cdot \frac{d\theta}{dt}$$
$$= [r \sin(\alpha - \theta)]\omega$$
$$= \omega r \sin(\alpha - \theta) \qquad (7.28)$$

It is maximum when $(\alpha - \theta)$ is maximum, i.e when θ is maximum or when the follower just touches the nose of the cam.

Velocity is minimum when $(\alpha - \theta)$ is minimum or when θ is maximum, i.e., when the follower is at the apex of the circular nose or $\theta = \alpha$.

$$f = \frac{dv}{dt} = \frac{dv}{d\theta} \cdot \frac{d\theta}{dt} = \omega r \cos(\alpha - \theta)(-1)\omega = -\omega^2 r \cos(\alpha - \theta) \qquad (7.29)$$

i.e., it is retardation. It is maximum when $(\alpha - \theta)$ is minimum or when θ is maximum or when the follower is at the apex of the nose.

Similarly, it is minimum at the commencement of the nose travel.

3. Circular Arc (Convex) Cam (with Roller Follower)

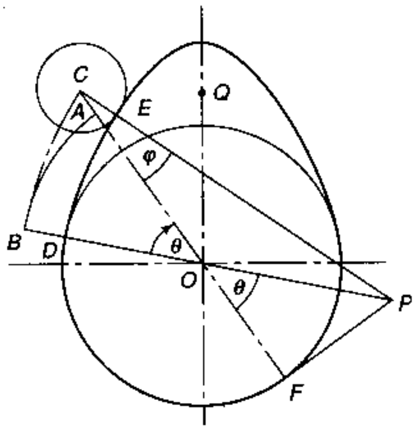

Fig. 7.35

Follower on the Flank (Fig. 7.35)

$$x = OC - OA = FC - FO - OB$$
$$= CP \cos\varphi - OP \cos\theta - OB$$
$$= CP \cos\varphi - (DP - DO) \cos\theta - OB$$

or

$$x = (r_f + r_r) \cos\varphi - (r_f + r_c) \cos\theta - (r_c + r_r)$$

where, $\cos\varphi$ is given by,

$$\cos\varphi = \sqrt{1 - \sin^2\varphi}$$
$$= \sqrt{1 - (FP / CP)^2}$$
$$= \sqrt{1 - \left(\frac{OP \sin\theta}{CP}\right)^2}$$

$$= \sqrt{1-\left[\frac{(r_f - r_c)\sin\theta}{(r_f + r_r)}\right]^2}$$

or $$\cos\varphi = \sqrt{1-\left(\frac{A\sin\theta}{B}\right)^2} = \frac{1}{B}\sqrt{B^2 - A^2\sin^2\theta}$$

where $A = r_f - r_c$ and $B = r_f + r_r$

$$x = \sqrt{B^2 - A^2\sin^2\theta} - A\cos\theta - (r_c + r_r) \quad (7.30)$$

$$v = \frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt} = \left[\frac{1}{2}(B^2 - A^2\sin^2\theta)^{-1/2}(-A^2 2\sin\theta\cos\theta) + A\sin\theta\right]\omega$$

$$= \omega A\left[\sin\theta - \frac{A\sin 2\theta}{2\sqrt{B^2 - A^2\sin^2\theta}}\right] \quad (7.31)$$

$$f = \frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt}$$

$$= \omega^2 A\left[\cos\theta - \frac{A}{2}\left\{\frac{2\cos 2\theta}{\sqrt{B^2 - A^2\sin^2\theta}} + \sin 2\theta\left(-\frac{1}{2}\right)(B^2 - A^2\sin^2\theta)^{-3/2}(-A^2)(2\sin\theta\cos\theta)\right\}\right]$$

$$= \omega^2 A\left[\cos\theta - \frac{A\cos 2\theta}{\sqrt{B^2 - A^2\sin^2\theta}} - \frac{A^3\sin 2\theta}{4(B^2 - A^2\sin^2\theta)^{3/2}}\right] \quad (7.32)$$

Follower on the Nose This case has already been discussed for the tangent cam when the roller follower is on the nose. Same expressions for the displacement, velocity and the acceleration hold good.

Example 7.8 *A tangent cam with straight working faces tangential to a base circle of 120 mm diameter has a roller follower of 48-mm diameter. The line of stroke of the roller follower passes through the axis of the cam. The nose circle radius of the cam is 12 mm and the angle between the tangential faces of the cam is 90°. If the speed of the cam is 180 rpm, determine the acceleration of the follower when*

(i) during the lift, the roller just leaves the straight flank

(ii) the roller is at the outer end of its lift, i.e., at the top of the nose

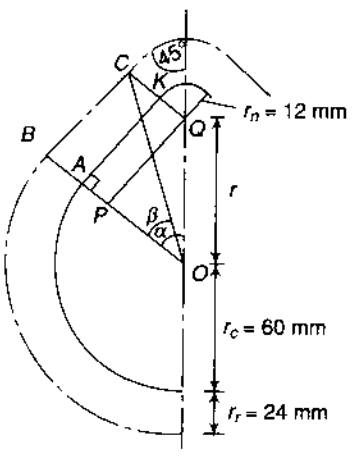

Fig. 7.36

Solution:

r_c = 60 mm $\quad r_n$ = 12 mm

r_r = 24 mm $\quad N$ = 180 rpm

$$\omega = \frac{2\pi \times 180}{60} = 6\pi \text{ rad/s}$$

Refer Fig. 7.36,

$$\alpha = (180° - 45° - 90°) = 45°$$

$$OA = OP + PA$$

$$= OQ \cos\alpha + QK$$

$$r_c = r\cos\alpha + r_n$$

$$60 = r\cos 45° + 12$$

$$r = 67.9 \text{ mm}$$

$$\tan\beta = \frac{BC}{OB} = \frac{PQ}{OB}$$

$$= \frac{r\sin\alpha}{r_c + r_r}$$

$$= \frac{67.9\sin 45°}{60 + 24}$$

$$= 0.571$$

$$\therefore \quad \beta = 29.74°$$

(i) Acceleration when the roller just leaves the straight flank,

$$f = \frac{\omega^2(r_c + r_r)\ (2 - \cos^2\theta)}{\cos^3\theta}$$

$$= \frac{(6\pi)^2\ (0.06 + 0.024)\ (2 - \cos^2 29.74°)}{\cos^3 29.74°}$$

$= (6\pi)^2 \times 0.084 \times 1.9035 = \underline{56.8 \text{ m/s}^2}$

(ii) Acceleration when the roller is at the outer end of its lift, i.e., at the top of the nose.

$\theta = \alpha$

$$f = \omega^2 r\left[\begin{array}{l} -\cos(\alpha - \theta) \\ -\dfrac{r^3\sin^2 2(\alpha-\theta)}{4\left[l^2 - r^2\sin^2(\alpha-\theta)\right]^{3/2}} \\ -\dfrac{r\cos 2(\alpha-\theta)}{\sqrt{l^2 - r^2\sin^2(\alpha-\theta)}} \end{array}\right]$$

$$= \omega^2 r\left[-1 - \frac{r}{l}\right]$$

$$= (6\pi)^2 \times 0.0679\left[-1 - \frac{0.0679}{0.036}\right]$$

$(l = r_r + r_n = 24 + 12 = 36 \text{ mm})$

$= \underline{69.6 \text{ m/s}^2}$

Example 7.9 ***A tangent cam with a base circle diameter of 50 mm operates a roller follower 20 mm in diameter. The line of stroke of the roller follower passes through the axis of the cam. The angle between the tangential faces of the cam is 60°, speed of the cam shaft is 200 rpm and the lift of the follower is 15 mm. Calculate the***

(i) main dimensions of the cam

(ii) acceleration of the follower at

(a) the beginning of lift

(b) where the roller just touches the nose

(c) the apex of the circular nose

Solution:

$r_c = 25$ mm $\quad h = 15$ mm

$r_r = 10$ mm $\quad N = 200$ rpm

$\alpha = (180° - 30° - 90°) = 60°$

(i) Refer Fig.7.37.

$r + r_n + r_r = r_c + r_r + h$

$r + r_n = r_c + h = 25 + 15 = 40$ (i)

Also, $OP + r_n = r_c$

or $r\cos 60° + r_n = 25$

or $0.5\,r + r_n = 25$ (ii)

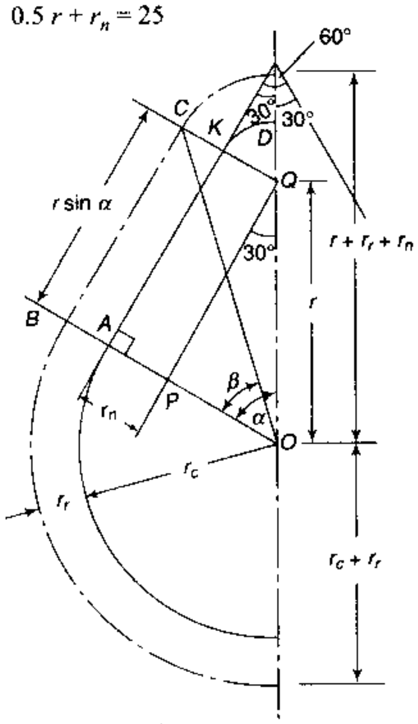

Fig. 7.37

Subtracting (ii) from (i),

$$r = 30 \text{ mm}$$
$$r_n = 10 \text{ mm}$$

$$\tan\beta = \frac{r\sin\alpha}{r_c + r_r} = \frac{30\sin 60^\circ}{25+10} = 0.742$$

$\therefore$ $\beta = 36.6^\circ$ or $36^\circ 36'$

(ii) Acceleration of the follower

$$\omega = \frac{2\pi \times 200}{60} = 20.94 \text{ rad/s}$$

(a) *At the beginning of Lift*, i.e., roller centre at B, $\theta = 0^\circ$.

$$f = \frac{\omega^2 (r_c + r_r)\,(2-\cos^2\theta)}{\cos^3\theta}$$
$$= \frac{(20.94)^2\,(0.025+0.01)\,(2-\cos^2 0^\circ)}{\cos^3 0^\circ}$$
$$= \underline{15.35 \text{ m/s}^2}$$

(b) *The roller just touches the nose* ($\theta = \beta$), i.e., the roller centre at C.

When the contact is with straight flank, $\theta = 36.6^\circ = \beta$

$$f = \frac{(20.94)^2\,(0.025+0.01)\,(2-\cos^2 36.6^\circ)}{\cos^3 36.6^\circ}$$
$$= \underline{40.2 \text{ m/s}^2}$$

When the contact is on the circular nose,

$l = r_n + r_r = 10 + 10 = 20$ mm

$$f = \omega^2 r \left[-\cos(\alpha-\theta) - \frac{r^3\sin^2 2(\alpha-\theta)}{4\left[l^2 - r^2\sin^2(\alpha-\theta)\right]^{3/2}} - \frac{r\cos 2(\alpha-\theta)}{\sqrt{l^2 - r^2\sin^2(\alpha-\theta)}} \right]$$

$$= (20.94)^2 (0.03) \left[-\cos(60^\circ - 36.6^\circ) - \frac{(0.03)^3\sin^2 2(60^\circ-36.6^\circ)}{4\left[(0.02)^2 - (0.03)^2\sin^2(60^\circ-36.6^\circ)\right]^{3/2}} - \frac{(0.03)\cos 2(60^\circ-36.6^\circ)}{\sqrt{(0.02)^2 - (0.03)^2\sin^2(60^\circ-36.6^\circ)}} \right]$$

$$= 13.15\,(-0.917 - 0.865 - 1.278) = \underline{-40.24 \text{ m/s}^2}$$

(c) *When the roller is at the apex of the circular nose*, i.e., at D, $\theta = \alpha$

and

$$f = \omega^2 r\left(-1 - \frac{r}{l}\right)$$
$$= (20.94)^2 \times 0.03\left(-1 - \frac{0.03}{0.02}\right) = \underline{-32.9 \text{ m/s}^2}$$

Example 7.10 ***The following data relate to a circular cam operating a flat-faced follower:***

Least diameter	=	*40 mm*
Lift	=	*12 mm*
Angle of action	=	*160°*
Speed	=	*500 rpm*

If the period of acceleration of the follower is 60° of the retardation during the lift, determine the

(i) main dimensions of the cam

(ii) acceleration at the main points

What is the maximum acceleration and deceleration during the lift?

Solution:

$r_c = 20$ mm $h = 15$ mm

$2\alpha = 160^\circ$ $N = 500$ rpm

$\alpha = 80^\circ$

During the lifting of the follower, the acceleration takes place when the follower is on the radial flank and the deceleration when the follower is on the nose. When the follower just touches the nose, the follower position will be as shown in Fig. 7.38. OC and PQE are parallel and the angles $\beta = 30^\circ$ and $\varphi = 50^\circ$ so that β is 60% of φ.

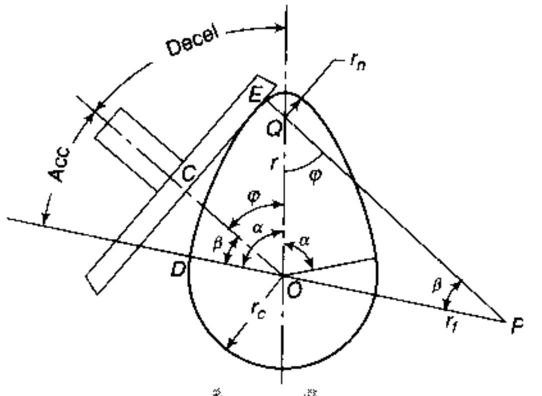

Fig. 7.38

(i) Apply sine rule to ΔPOQ,

$$\frac{OP}{\sin \varphi} = \frac{PQ}{\sin (180° - \alpha)} = \frac{OQ}{\sin \beta}$$

$$\frac{r_f - r_c}{\sin 50°} = \frac{r_f - r_n}{\sin 100°} = \frac{r}{\sin 30°}$$

But $r + r_n = r_c + h$

or $r = r_c + h - r_n$

$= 20 + 12 - r_n$

$= 32 - r_n$

$$\frac{r_f - 20}{\sin 50°} = \frac{r_f - r_n}{\sin 100°} = \frac{32 - r_n}{\sin 30°}$$

From first and last terms,

$0.5\, r_f - 10 = 24.51 - 0.766\, r_n$

$0.5\, r_f = 34.51 - 0.766\, r_n$

or $r_f = 69.02 - 1.532\, r_n$ (i)

From second and last terms,

$0.5\, r_f - 0.5\, r_n = 31.51 - 0.985\, r_n$

$r_f = 63.02 - 0.97\, r_n$ (ii)

From (i) and (ii),

$69.02 - 1.532\, r_n = 63.02 - 0.97\, r_n$

$r_n = \underline{10.7 \text{ mm}}$

$r = 32.0 - 10.7 = \underline{21.3 \text{ mm}}$

$r_f = 69.02 - 1.532 \times 10.7 = \underline{52.6 \text{ mm}}$

(ii) Accelerations

(a) At the beginning of contact, $\theta = 0°$,

$$f = \omega^2 (r_f - r_c) \cos 0°$$

or $$f = \left(\frac{2\pi \times 500}{60} \right)^2 (52.6 - 20)$$

$= 2742 \times 32.6$

$= 89\,370 \text{ mm/s}^2$ or 89.37 m/s^2

(b) Contact on circular flank when $\theta = \beta = 30°$,
$f = 2742 \times 32.6 \cos 30° = 77\,410 \text{ mm/s}^2$ or 77.41 m/s^2

(c) Contact on circular nose, when $\theta = \beta = 30°$,
$f = -\omega^2 r \cos (\alpha - \theta) = -2742 \times 21.3 \cos (80° - 30°)$
$= -37\,540 \text{ mm/s}^2$ or -37.54 m/s^2

(d) Contact at the apex of nose, $\theta = \alpha = 80°$,

$f = -\omega^2 r \cos (80° - 80°) = -2742 \times 21.3$

$= -58\,400 \text{ mm/s}^2$ or -58.4 m/s^2

Maximum acceleration is when the contact is just made with the circular flank; it is 89.37 m/s^2 and the maximum retardation is at the end of the lifting period, i.e., when the contact is at the apex of the nose; it is 58.4 m/s^2.

Example 7.11 *The following data relate to a symmetrical circular cam operating a flat-faced follower:*

Minimum radius of the cam	*40 mm*
Lift	*24 mm*
Angle of lift	*75°*
Nose radius	*8 mm*
Speed of the cam	*420 rpm*

Determine the main dimensions of the cam and the acceleration of the follower at the

(i) beginning of the lift

(ii) end of contact with the circular flank

(iii) beginning of contact with the nose

(iv) apex of nose

Solution

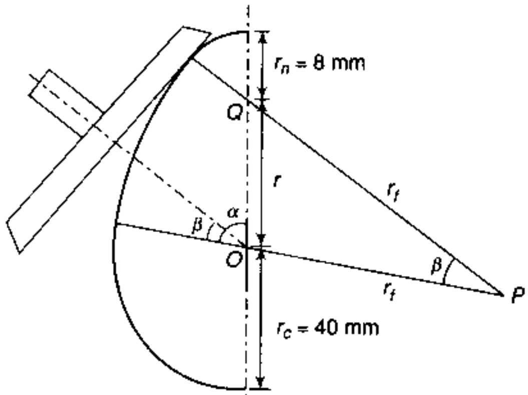

Fig. 7.39

$r_c = 40$ mm $\quad N = 400$ rpm

$h = 24$ mm $\quad r_n = 8$ mm

$\alpha = 75°$

$$\omega = \frac{2\pi \times 420}{60} = 44 \text{ rad/s}$$

Refer to Fig.7.39.

$r + r_n = r_c + h$

or $r = 40 + 24 - 8 = 56$ mm

$(PQ)^2 = (OP)^2 + (OQ)^2 - 2(OP)(OQ) \cos \angle POQ$

$(r_f - 8)^2 = (r_f - 40)^2 + (56)^2 - 2(r_f - 40)(56)\cos(180° - 75°)$

$$r_f^2 + 64 - 16r_f = r_f^2 + 1600 - 80r_f + 3136 + 29r_f - 1160$$

$35\,r_f = 3512$

$r_f = 100.3$ mm

$OP = 100.3 - 40 = 60.3$ mm

$PQ = 100.3 - 8 = 92.3$ mm

Applying sine rule to ΔOPQ,

$$\frac{OQ}{\sin\beta} = \frac{PQ}{\sin(180° - \alpha)}$$

or $$\frac{r}{\sin\beta} = \frac{r_f - r_n}{\sin 105°}$$

or $$\frac{56}{\sin\beta} = \frac{100.3 - 8}{\sin 105°}$$

$\sin\beta = 0.586$

$\beta = 35.9°$

Acceleration when the follower is on the circular flank, $f = \omega^2 (r_f - r_c)\cos\theta$

(i) At the beginning of lift, $\theta = 0°$,

$f = \omega^2 (r_f - r_c) = 44^2\,(100.3 - 40)$
$= 116\,740$ mm/s^2 = 116.74 m/s^2

(ii) At the end of contact with the circular flank,

$f = \omega^2 (r_f - r_c)\cos\theta = 44^2\,(100.3 - 40)\cos 35.9° = 94\,565$ mm/s^2 = 94.565 m/s^2

Acceleration when the follower is on the nose, $f = -\omega^2 r\cos(\alpha - \beta)$

(iii) At the beginning of contact with the nose

$f = -\omega^2 r\cos(\alpha - \beta) = -44^2 \times 56 \times \cos(75° - 35.9°) = -84\,136$ mm/s^2
or -84.136 m/s^2

(iv) At the apex of the nose, $\alpha = \beta$

$f = -\omega^2 r = -44^2 \times 56 = -108\,416$ mm/s^2
or -108.416 m/s^2

Example 7.12 *In a four-stroke petrol engine, the exhaust valve opens 45° before the t.d.c. and closes 15° after the b.d.c. The valve has a lift of 12 mm. The least radius of the circular-arc-type cam operating a flat-faced follower is 25 mm. The nose radius is 3 mm. The camshaft rotates at 1500 rpm. Calculate the maximum velocity of the valve and the minimum force exerted by the spring to overcome the inertia of the moving parts that weigh 300 g.*

Solution:

$r_c = 25$ mm $N = 1500$ rpm
$h = 12$ mm $r_n = 3$ mm
$m = 0.3$ kg

Crank rotation during of the exhaust valve = 45° + 180° + 15° = 240°

In four-stroke engines, the camshaft speed is half that of the crankshaft.

Angle of action of the camshaft,

$$2\alpha = \frac{240}{2} = 120°$$

$$\therefore \quad \alpha = \frac{120}{2} = 60°$$

Refer to Fig. 7.37,

$r + r_n = r_c + h$

or $r = 25 + 12 - 3 = 34$ mm

$(PQ)^2 = (OP)^2 + (OQ)^2 - 2\,(OP)\,(OQ)\cos\angle POQ$

$(r_f - 3)^2 = (r_f - 25)^2 + (34)^2 - 2\,(r_f - 25)\,(34)\cos(180° - 60°)$

$$r_f^2 + 9 - 6r_f = r_f^2 + 625 - 50r_f + 1156 + 34r_f - 850$$

$r_f = 92.2$ mm

Applying sine rule to ΔOPQ,

$$\frac{OQ}{\sin\beta} = \frac{PQ}{\sin(180° - \alpha)}$$

$$\frac{r}{\sin\beta} = \frac{r_f - r_n}{\sin 120°}$$

$$\frac{34}{\sin\beta} = \frac{92.2 - 3}{\sin 120°}$$

$\sin\beta = 0.33$

$\alpha = 19.27°$

Velocity is maximum when the contact is on the point where the circular flank meets the circular nose.

$$v_{max} = \omega (r_f - r_c)\sin\beta$$

$$= \frac{2\pi \times 1500}{60}(92.2 - 25)\sin 19.27°$$

$= 157.08 \times 67.2 \times 0.33$

$= 3480$ mm/s or 3.48 m/s

The same result is obtained if relation of Eq. (7.28) is used, i.e.,

$$v_{max} = \omega r \sin(\alpha - \theta)$$

Maximum acceleration is when $\theta = 0$,

$f_{max} = \omega^2 (r_f - r_c) = (157.08)^2 (92.2 - 25)$
$= 1658\ 090$ mm/s² or 1658.09 m/s²

Maximum retardation is when $\alpha - \theta = 0$,

$f_{max} = \omega^2 r = (157.08)^2 \times 34 = 838\ 920$ mm/s² or 838.92 m/s²

Spring force is needed to maintain contact during the retardation of the follower.

Minimum force, $F = m \times f$
$= 0.3 \times 838.92 = 251.7$ N

7.11 ANALYSIS OF A RIGID ECCENTRIC CAM

Analysis of a rigid eccentric cam involves the determination of the contact force, the spring force and the cam shaft torque for one revolution of the cam. In simplified analysis, all the components of the cam system are assumed to be rigid and the results are applicable to low-speed systems. However, if the speeds are high and the members are elastic, an elastic body analysis must be made. The elasticity of the members may be due to extreme length of the follower or due to use of elastic materials in the system. In such cases, noise, excessive wear, chatter, fatigue failure of some of the parts are the usual things.

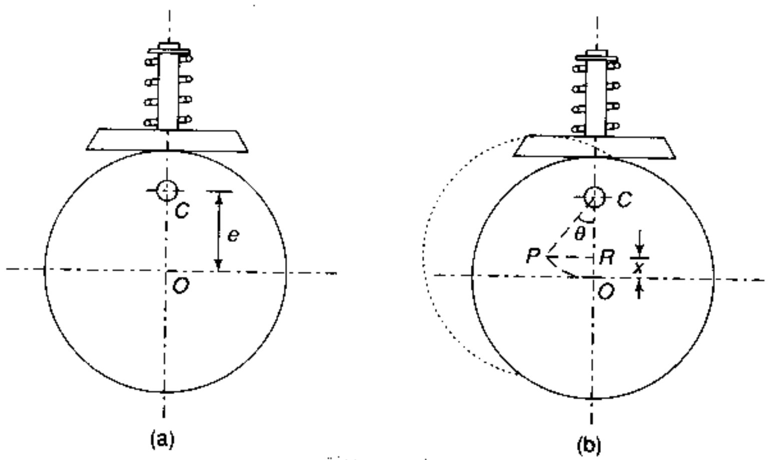

Fig. 7.40

A circular disc cam with the cam shaft hole drilled off centre is known as an *eccentric plate cam*. Figure 7.40(a) shows a simplified reciprocating eccentric cam system consisting of a plate cam, a flat face follower and a retaining spring.

Let e = eccentricity, the distance between the centre of the disc and of the shaft
m = mass of the follower
s = stiffness of the retaining spring
ω = angular velocity of the cam rotation
P = preload including the weight of the follower or the force on the cam at $x = 0$
x = motion of the follower (zero at the bottom of the stroke)

If the disc is rotated through an angle θ [Fig. 7.40(b)], the mass m is displaced by a distance x, so that

$$x = e - e\cos\theta \tag{7.33}$$

Velocity,
$$\dot{x} = \frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt} = e\omega\sin\theta \tag{7.34}$$

Acceleration,
$$\ddot{x} = \frac{d\dot{x}}{dt} = \frac{d\dot{x}}{d\theta}\frac{d\theta}{dt} = e\omega^2\cos\theta \tag{7.35}$$

Now, as the follower is displaced in the upward direction, the acceleration $\ddot{x}$ of the follower is in the upward direction indicating the inertia force to be in the downward direction. The spring force is also exerted in the downward direction. The force exerted by the cam on the follower F, however, will be in the upward direction.

Thus, the various forces acting on the follower mass are

Inertia force	$= m\ddot{x}$	(downwards)
Spring force	$= sx$	(downwards)
Preload	$= P$	(downwards)
Force exerted by cam	$= F$	(upwards)

Thus the equilibrium equation becomes,

$$m\ddot{x} + sx + P - F = 0$$

$$F = m\ddot{x} + sx + P$$

$$= me\omega^2 \cos \omega t + se - se \cos \omega t + P$$

$$= (se + P) + (m\omega^2 - s)\, e \cos \omega t \qquad (7.36)$$

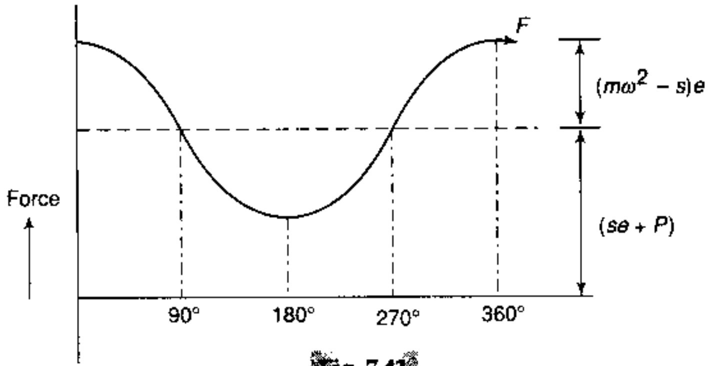

Fig. 7.41

Equation 7.36 shows that the force F exerted by a cam on the follower consists of a constant term $(se + P)$ with a cosine wave superimposed on it, the maximum value of which occurs at $\theta = 0°$ and minimum at $\theta = 180°$. Figure 7.41 shows the change of this force with the angular displacement of the cam. As the cam shaft velocity increases, the term involving square of the velocity increases at a faster rate and the force F becomes zero at a speed when

$$(se + P) + (m\omega^2 - s)\, e \cos \omega t = 0 \qquad (7.37)$$

This can happen at $\theta = 180°$. At that speed, there will be some impact between the cam and the follower, resulting in a rattling, clicking and noisy operation. This is usually known as *jump*. However, this can be prevented to some extent by increasing the preload exerted by the spring or the spring stiffness.

At jump speed,

$$(se + P) - (m\omega^2 - s)\, e = 0$$

or

$$2se + P - me\omega^2 = 0$$

or

$$\omega = \sqrt{\frac{2se + P}{me}} \qquad (7.38)$$

and jump will not occur if preload of the spring is increased such that $P > e(m\omega^2 - 2s)$

The torque applied by the shaft to the cam,

$$T = F.\, e \sin \omega t$$

$$= [(se + P) + (m\omega^2 - s)\, e \cos \omega t]\, e \sin \omega t$$

$$= e(se + P) \sin \omega t + \frac{e^2}{2}(m\omega^2 - s) \sin 2\omega t \qquad (7.39)$$

Figure 7.42 shows the variation of torque with the cam rotation. It may be observed from the plots that area of the torque- displacement diagram above and below the X-axis is the same meaning that the energy required to raise the follower is recovered during the return. A flywheel may be used to handle this fluctuation of energy.

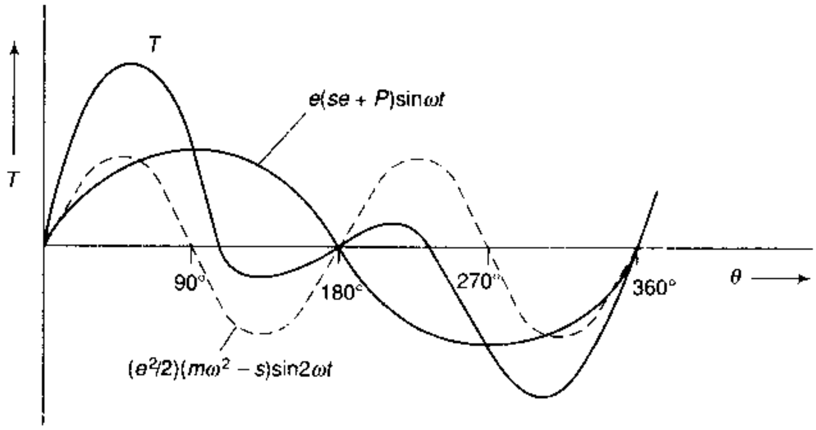

Fig. 7.42

Example 7.13 *A circular disc cam of 120 mm diameter with its centre displaced at 40 mm from the camshaft is used with a flat surface follower. The line of action of the follower is vertical and passes through the shaft axis. The mass of the follower is 3 kg and is pressed downwards with a spring of stiffness 5 N/mm. In the lowest position, the spring force is 60 N.*

Derive an expression for the acceleration of the follower as a function of cam rotation from the lowest position of the follower. Also, find the speed at which the follower begins to lift from the cam surface.

Solution:

$e = 40$ mm $\qquad m = 3$ kg

$s = 5$ N/mm $= 5\,000$ N/m

$P = 60 \text{ N} + \text{mg} = (60 + 3 \times 9.81)$ N

Consider the rotation of the cam through angle θ (Refer Fig. 7.43),

Now, $x = 40 - 40\, e \cos\theta = 40\,(1 - \cos\theta)$

$$\dot{x} = \frac{dx}{dt} = \frac{dx}{d\theta}\frac{d\theta}{dt} = 40\omega \sin\theta$$

$$\ddot{x} = \frac{dv}{dt} = \frac{dv}{d\theta}\frac{d\theta}{dt} = 40\omega^2 \cos\theta$$

which is the required expression for acceleration of the cam follower system.

To find the speed at which the follower begins to lift from the cam surface or the jump speed,

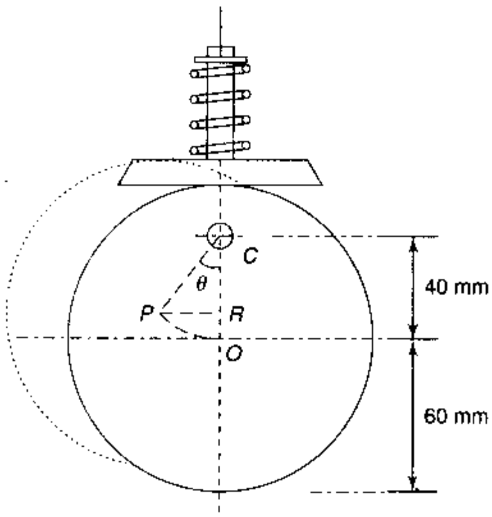

Fig. 7.43

$$\omega = \sqrt{\frac{2se + P}{me}} \qquad \text{(Eq. 7.38)}$$

$$= \sqrt{\frac{2 \times 5000 \times 0.04 + 60 + 3 \times 9.81}{3 \times 0.04}}$$

$$= \sqrt{4078.6}$$

$$= 63.86 \text{ rad/s}$$

or $\frac{2\pi N}{60} = 63.86$

or $N = \underline{609.9 \text{ rpm}}$

7.12 ANALYSIS OF AN ELASTIC CAM SYSTEM

To illustrate the effect of follower elasticity upon its displacement and velocity, consider a simplified model of a cam system with a linear motion used for low-speed cams (Fig. 7.44).

Let

m = lumped mass of the follower
s_1 = stiffness of the retaining spring
s_2 = stiffness of the follower
x = displacement of the lumped mass of the follower
y = motion machined into the cam surface
h = lift of the follower

Assume a cam profile that gives a uniform rise for an angle of rotation ϕ followed by a dwell. Thus,

$$y = h\frac{\theta}{\varphi} = h\frac{\omega t}{\varphi}$$

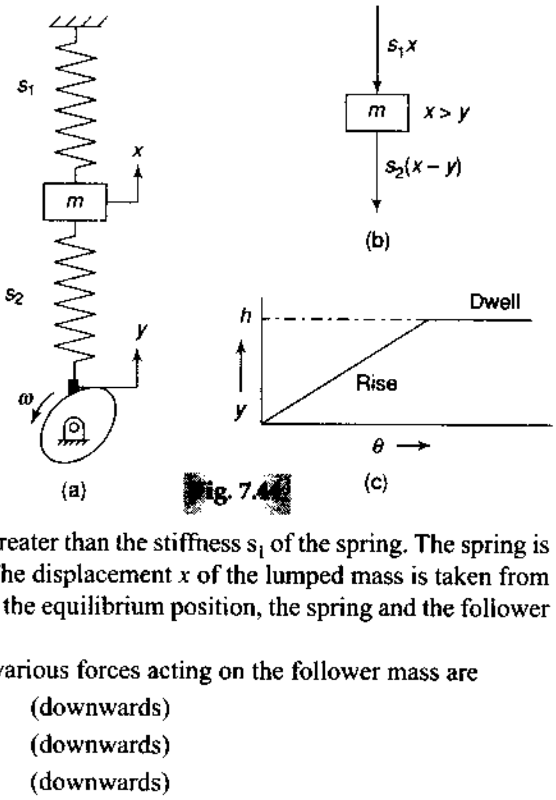

Fig. 7.44

As the follower is usually a rod, its stiffness s_2 is far greater than the stiffness s_1 of the spring. The spring is assembled in such a way that it exerts a preload force. The displacement x of the lumped mass is taken from the equilibrium position after the spring is assembled. In the equilibrium position, the spring and the follower exert equal and opposite preload forces on the mass.

Assuming the displacement x to be more than y, the various forces acting on the follower mass are

Inertia force = $m\ddot{x}$ (downwards)

Spring force = s_1x (downwards)

Force of elastic follower = $s_2(x - y)$ (downwards)

Thus

$$m\ddot{x} + s_1x_1 + s_2(x - y) = 0$$

$$\ddot{x} + \frac{s_1 + s_2}{m}x = \frac{s_2}{m}y$$

$$\ddot{x} + \omega_n^2 x = \frac{s_2}{m}y$$

During ascent, the displacement of the follower mass is given by the solution of the above equation, i.e.,

$$x = A\cos\omega_n t + B\sin\omega_n t + \frac{s_2}{m\omega_n^2}y \tag{i}$$

Differentiating with respect to t,

$$\dot{x} = -A\omega_n\sin\omega_n t + B\omega_n\cos\omega_n t + \frac{s_2}{m\omega_n^2}\dot{y} \tag{ii}$$

When $t = 0$, $x = \dot{x} = 0$ and $y = 0$

$\therefore$ from (i), $0 = A + 0 + 0$

or $A = 0$

From (ii),

$$0 = 0 + B\omega_n + \frac{s_2}{m\omega_n^2}\dot{y}$$

$$B = -\frac{s_2}{m\omega_n^2}\dot{y}$$

$\therefore$ (i) becomes,

$$x = 0 - \frac{s_2}{m\omega_n^2}\dot{y}\sin\omega_n t + \frac{s_2}{m\omega_n^2}y \tag{7.40}$$

The particular integral $\frac{s_2}{m\omega_n^2}y$ is called the *follower command*.

$$x = \frac{s_2}{m\omega_n^2}\left(y - \frac{\dot{y}}{\omega_n}\sin\omega_n t\right)$$

$$= \frac{s_2}{m\omega_n^2}\left(\frac{h}{\varphi}\theta - \frac{h\omega}{\varphi\omega_n}\sin\omega_n t\right) \quad \cdots\left(\dot{y} = \frac{h}{\varphi}\omega\right)$$

$$\therefore \quad x = \frac{s_2 h}{m\omega_n^2\varphi}\left(\omega t - \frac{\omega}{\omega_n}\sin\omega_n t\right)$$

$$= \frac{\omega \cdot s_2 h}{m\omega_n^2\varphi}\left(t - \frac{\sin\omega_n t}{\omega_n}\right)$$

$$= \frac{s_2 h}{rm\omega_n\varphi}\left(t - \frac{\sin\omega_n t}{\omega_n}\right)$$

where $r = \omega_n/\omega$

$$\dot{x} = \frac{s_2 h}{rm\omega_n\varphi}\left(1 - \frac{\omega_n}{\omega_n}\cos\omega_n t\right)$$

$$= \frac{s_2 h}{rm\omega_n\varphi}(1 - \cos r\theta)$$

At the peak,

$$x = x_1, \; \theta = \varphi$$

$$x_1 = \frac{s_2 h}{rm\omega_n^2\varphi}(r\varphi - \sin r\varphi)$$

$$\dot{x}_1 = \frac{s_2 h}{rm\omega_n\varphi}(1 - \cos\varphi)$$

These become the initial conditions for dwell period.

For dwell period,

$$x = A\cos\omega_n t + B\sin\omega_n t + \frac{s_2}{m\omega_n^2}h$$

$$\dot{x} = -A\omega_n \sin \omega_n t + B\omega_n \cos \omega_n t \qquad \left(\frac{s_2}{m\omega_n^2} h \text{ is constant} \right)$$

At $t = 0$, $x = x_1$, $y = h$, $\dot{x} = \dot{x}_1$

$$x_1 = A + \frac{s_2 h}{m\omega_n^2}, \qquad \therefore \qquad A = x_1 - \frac{s_2 h}{m\omega_n^2},$$

$$\dot{x}_1 = B\omega_n, \qquad \therefore \qquad B = \frac{\dot{x}_1}{\omega_n}$$

Therefore, during the dwell period,

$$x = \left(x_1 - \frac{s_2 h}{m\omega_n^2} \right) \cos \omega_n t + \frac{\dot{x}_1}{\omega_n} \sin \omega_n t + \frac{s_2}{m\omega_n^2} h \qquad (7.41)$$

Figure 2.45 shows the response of the follower x, the cam motion y, and the follower command. The follower command is different from the cam motion due to the elasticity of the follower.

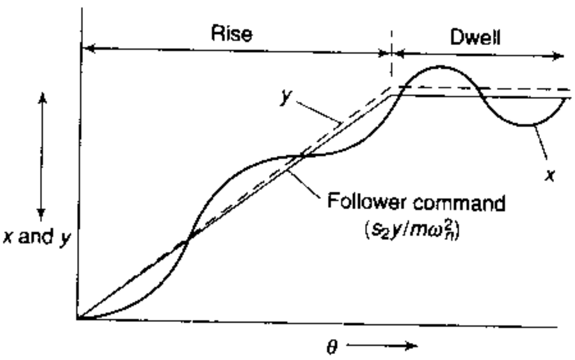

Fig. 7.45

7.13 SPRING SURGE, UNBALANCE AND WIND UP

Usually, the helical springs have a tendency to vibrate of their own when subjected to rapidly varying forces. This vibration of the retaining spring of a cam is called *spring surge*. Valve springs of the automobiles that operate near the critical frequency may keep the valve open for short periods when they are supposed to be closed. This results in lowering the efficiency of the engine apart from early fatigue failure of the springs.

If the mass of the cam is not uniformly distributed about the axis of the rotation, *unbalance* is produced on the shaft. This, along with the reaction of the follower against the cam, produce vibratory forces in a cam. Therefore, a face or end cam has better balance characteristics as compared to a disc cam.

It is observed from the plot of cam shaft torque (Fig.7.42) that it varies over a complete rotation of the cam. This tends to twist or *wind up* the shaft. During the rise of the follower, the angular cam velocity is decreased which also results in the slower follower velocity. As the follower reaches near the end of rise, the energy stored as a result of wind up is released resulting in velocity and acceleration of the follower to rise above average values. This may produce follower *jump* or impact. This phenomenon is more pronounced during the movement of heavy loads by the follower, when the follower moves at high speed or when the shaft is flexible.

Summary

1. A cam is a mechanical member used to impart desired motion to a follower by direct contact.
2. Complicated output motions which are otherwise difficult to achieve can easily be produced with the help of cams.
3. Cams are classified according to shape, follower movement, and the manner of constraint of the follower.
4. The motions of the followers are distinguished from each other by the dwells they have. A dwell is the zero displacement or the absence of motion of the follower during the motion of the cam.
5. Cam followers are classified according to shape, movement, and the location of line of movement.
6. *Base circle* is the smallest circle tangent to the cam profile (contour) drawn from the centre of rotation of a radial cam.
7. Pitch curve is the curve drawn by the trace point assuming the cam to be fixed and rotating the trace point of the follower around the cam.
8. The pressure angle, representing the steepness of the cam profile, is the angle between the normal to the pitch curve at a point and the direction of the follower motion. It varies in magnitude at all instants of the follower motion.
9. In simple harmonic motion, the maximum velocity of the follower is $v_{max} = \frac{h}{2}\frac{\pi\omega}{\varphi}$ at $\theta = \frac{\varphi}{2}$ and maximum acceleration
$$f_{max} = \frac{h}{2}\left(\frac{\pi\omega}{\varphi}\right)^2 \text{ at } \theta = 0^\circ.$$
10. In constant acceleration and deceleration, acceleration is given by, $f = \frac{4h\omega^2}{\varphi^2}$ and maximum velocity $v_{max} = \frac{2h\omega}{\varphi}$ at $\theta = \varphi/2$.
11. In constant velocity of the follower, the constant velocity is given by $v = \frac{h\omega}{\varphi}$.
12. In cycloidal motion, $v_{max} = \frac{2h\omega}{\varphi}$ at $\theta = \frac{\varphi}{2}$ and $f_{max} = \frac{2h\pi\omega^2}{\varphi^2}$ at $\theta = \frac{\varphi}{4}$
13. Cycloidal motion is the most ideal programme for high-speed follower motion.
14. A tangent cam is symmetrical about the centre line. It has straight flanks with circular nose.
15. A circular arc cam is made up of three arcs of different radii. In such cams, the acceleration may change abruptly at the blending points due to instantaneous change in the radius of curvature.

Exercises

1. What is a cam? What type of motion can be transmitted with a cam and follower combination? What are its elements?
2. How are the cams classified? Describe in detail.
3. Discuss various types of cams.
4. Compare the performance of knife-edge, roller and mushroom followers.
5. Define base circle, pitch circle, trace point, pitch curve and pressure angle.
6. What are the requirements of a high-speed cam?
7. Which follower programme do you recommend for a high-speed cam and why?
8. What is a displacement diagram? Why is it necessary to draw it before drawing a cam profile?
9. Deduce expressions for the velocity and acceleration of the follower when it moves with simple harmonic motion.
10. Why is a cycloidal motion programme most suitable for high-speed cams?
11. Explain the procedure to lay out the cam profile for a reciprocating follower.
12. What is a tangent cam? Find the expressions for the velocity and acceleration of a roller follower for such a cam.
13. Derive relations for velocity and acceleration for a convex cam with a flat-faced follower.
14. Draw the profile of a cam that gives a lift of 40 mm to a rod carrying a 20 mm diameter roller. The axis of the roller passes through the centre of the cam. The least radius of the cam is 50 mm. The rod is to be lifted with simple harmonic motion in a quarter revolution and is to be dropped suddenly at half

revolution. Determine the maximum velocity and maximum acceleration during the lifting. The cam rotates at 60 rpm.

(0.25 m/s; 3.155 m/s²)

15. Lay out the profile of a cam so that the follower
 - is moved outwards through 30 mm during 180° of cam rotation with cycloidal motion
 - dwells for 20° of the cam rotation
 - returns with uniform velocity during the remaining 160° of the cam rotation

 The base circle diameter of the cam is 28 mm and the roller diameter 8 mm. The axis of the follower is offset by 6 mm to the left. What will be the maximum velocity and acceleration of the follower during the outstroke if the cam rotates at 1500 rpm counter-clockwise?

 (3 m/s; 471.2 m/s²)

16. Use the following data in drawing the profile of a cam in which a knife-edged follower is raised with uniform acceleration and deceleration and is lowered with simple harmonic motion:

 Least radius of cam = 60 mm
 Lift of follower = 45 mm
 Angle of ascent = 60°
 Angle of dwell between ascent and descent = 40°
 Angle of descent = 75°

 If the cam rotates at 180 rpm, determine the maximum velocity and acceleration during ascent and descent.

 (*Ascent:* 1.62 m/s; 58.3 m/s²)
 Descent: 1.02 m/s; 46.05 m/s²)

17. Draw the profile of a cam which is to give oscillatory motion to the follower with uniform angular velocity about its pivot. The base circle diameter is 50 mm, angle of oscillation of the follower is 30° and the distance between the cam centre and the pivot of the follower is 60 mm. The oscillating lever is 60 mm long with a roller of 8-mm diameter at the end. One oscillation of the follower is completed in one revolution of the cam.

18. Set out the profile of a cam to give the following motion to a flat mushroom contact face follower:
 - Follower to rise through 24 mm during 150° of cam rotation with SHM
 - Follower to dwell for 30° of the cam rotation
 - Follower to return to the initial position during 90° of the cam rotation with SHM
 - Follower to dwell for the remaining 90° of cam rotation

 Take minimum radius of the cam as 30 mm.

19. A cam is required to give motion to a follower fitted with a roller that is 50 mm in diameter. The lift of the follower is 30 mm and is performed
 - with uniform acceleration for 12 mm, the cam turns through 45°
 - with uniform velocity for 12 mm, the cam turns through the next 30°
 - with uniform deceleration for the remainder of the lift, the cam turns through the next 45°

 The follower falls through immediately with simple harmonic motion while the cam turns through 120°. Then a period of dwell is followed for 120° of the cam angle. Construct a lift and fall diagram on a cam angle base. Also, draw the outline of the cam. The least radius of the cam is 35 mm. The line of motion of the follower passes through the centre of the cam axis.

20. Draw the profile of a cam operating a roller reciprocating follower having a lift of 35 mm. The line of stroke of the follower passes through the axis of the cam shaft. The radius of the roller is 10 mm and the minimum radius of the cam is 40 mm. The cam rotates at 630 rpm counter-clockwise. The follower is raised with simple harmonic motion for 90° of the cam rotation, dwells for next 60° and then lowers with uniform acceleration and deceleration for the next 150°. The follower dwells for the rest of the cam rotation.

 Also, draw the displacement, velocity and the acceleration diagrams for the motion of the follower for one complete revolution of the cam indicating main values.

 (2.31 m/s, 304.9 m/s², 5.54 m/s, 878.2 m/s²)

21. A tangent cam with straight working faces is tangential to a base circle of 80 mm diameter. It operates a roller follower of 32 mm diameter. The line of stroke of the follower passes through the axis of the cam. The nose circle radius of the cam is 10 mm and the angle between the tangential faces of the cam is 90°. If the speed of the cam is 315 rpm, determine the acceleration of the follower when (i) during the lift, the roller just leaves the straight flank, and (ii) the roller is at the outer end of its lift, i.e., at the top of the nose.

 (105.1 m/s², 799.1 m/s²)

22. The following particulars relate to a symmetrical tangent cam having a roller follower:

 Minimum radius of the cam = 40 mm
 Lift = 20 mm
 Speed = 360 rpm

Roller diameter = 44 mm
Angle of ascent = 60°

Calculate the acceleration of a follower at the beginning of the lift. Also, find its values when the roller just touches the nose and is at the apex of the circular nose. Sketch the variation of displacement, velocity and acceleration during ascent.

(88.12 m/s²; 164 m/s²; and –92.6 m/s²; –111 m/s²)

23. A flat-ended valve tappet is operated by a symmetrical cam with circular arcs for flank and nose profiles. The total angle of action is 150°, base circle diameter is 125 mm and the lift is 25 mm. During the lift, the period of acceleration is half that of the declaration. The speed of cam shift is 1250 rpm. The straight-line path of the tappet passes through the cam axis. Find
    (i) radii of the nose and the flank, and
    (ii) maximum acceleration and declaration during the lift.

(40.3 mm, 148 mm; 1465 m/s²; 808.8 m/s²)

24. In a four-stroke petrol engine, the crank angle is 5° after t.d.c. when the suction valve opens and 53° after b.d.c. when the suction valve closes. The lift is 8 mm, the nose radius is 3 mm and the least radius of the cam is 18 mm. The shaft rotates at 800 rpm. The cam is of the circular type with a circular nose and flanks while the follower is flat-faced. Determine the maximum velocity and the maximum acceleration and retardation of the valve. When is the minimum force exerted by the springs to overcome the inertia of moving parts weighting 250 g.

(1.3 m/s; 433.7 m/s; 161.4 m/s²; 40.35 N)

25. A symmetrical circular cam operates a flat-faced follower with a lift of 30 mm. The minimum radius of the cam is 50 mm and the nose radius is 12 mm. The angle of lift is 80°. If the speed of the cam is 210 rpm, find the main dimensions of the cam and the acceleration of the follower at (i) the beginning of the lift (ii) the end of contact with the circular flank (iii) the beginning of contact with the nose, and (iv) the apex of nose.

(r = 68 mm, 29.38 m/s², 23.8 m/s², 26.2 m/s², 32.9 m/s²)

26. A circular disc cam with diameter of 80 mm with its centre displaced at30 mm from the camshaft is used with a flat surface follower. The line of action of the follower is vertical and passes through the shaft axis. The mass of the follower is 2.5 kg and is pressed downwards with a spring of stiffness 4 N/mm. In the lowest position, the spring force is 50 N. Derive an expression for the acceleration of the follower as a function of cam rotation from the lowest position of the follower. Also, find the speed at which the follower begins to lift from the cam surface.

(618.4 rpm)

8 FRICTION

Introduction

When a body slides over another, the motion is resisted by a force called the *force of friction*. The force arises from the fact that the surfaces, though planed and made smooth, have ridges and depressions that interlock and the relative movement is resisted. Thus, the force of friction on a body is parallel to the sliding surfaces and acts in a direction opposite to that of the sliding body.

There are phenomena, where it is necessary to reduce the force of friction whereas in some cases it must be increased. In case of lathe slides, journal bearings, etc., where the power transmitted is reduced due to friction, it has to be decreased by the use of lubricated surfaces. In processes where the power is transmitted through friction, attempts are made to increase it to transmit more power. Examples are friction clutches and belt drives. Even the tightness of a nut and bolt is dependent mainly on the force of friction. Had there been no friction between the nut and the surface on which it is tightened, the nut would loosen off at the moment the spanner is removed after tightening.

8.1 KINDS OF FRICTION

Usually, three kinds of friction, depending upon the conditions of surfaces are considered.

1. Dry Friction

Dry friction is said to occur when there is relative motion between two completely unlubricated surfaces. It is further divided into two types:

(a) ***Solid Friction*** When the two surfaces have a sliding motion relative to each other, it is called a solid friction.

(b) ***Rolling Friction*** Friction due to rolling of one surface over another (e.g., ball and roller bearings) is called rolling friction (Sec 8.10).

2. Skin or Greasy Friction

When the two surfaces in contact have a minute thin layer of lubricant between them, it is known as *skin* or *greasy friction*. Higher spots on the surface break through the lubricant and come in contact with the other surface.

Skin friction is also termed as *boundary* friction (Sec 8.12).

3. Film Friction

When the two surfaces in contact are completely separated by a lubricant, friction will occur due to the shearing of different layers of the lubricant. This is known as *film friction* or *viscous* friction (Sec. 8.15).